UNIVERSITY OF LJUBLJANA
SCHOOL OF ECONOMICS AND BUSINESS

MASTER’S THESIS

THE USE OF COMPUTATIONAL INTELLIGENCE TECHNIQUES FOR MID-TERM ELECTRICITY PRICE FORECASTING

Ljubljana, March 2021

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LIST OF ABBREVIATIONS

ARIMA	Autoregressive Integrated Moving Average
CI	Computational Intelligence
CRISP-DM	Cross-Industry Standard Process for Data Mining
DSO	Distribution System Operators
EPX	Electric Power Exchange
ESP	Electricity Spot Prices
ESPF	Electricity Spot Price Forecasting
ETS	Exponential Smoothing
EU	European Union
k-NN	K-Nearest Neighbor
LR	Linear Regression
MAE	Mean Absolute Error

MAPE	Mean Absolute Percentage Error
MCO	Market Coupling Operator
MCP	Market Clearing Price
MIBEL	Iberian Electricity Market
MSE	Mean Square Error
NEMO	Nominated Electricity Market Operator
NNAR	Autoregressive Neural Network
OMIE	Iberian Peninsula Nominated Electricity Market
PCR	Price Coupling of Regions
PRISMA	Preferred Reporting Items for Systematic Reviews
RF	Random Forest
RMSE	Root Mean Square Error
SVM	Support Vector Machine
TSO	Transmission System Operators
XGBoost	Extreme Gradient Boosting

INTRODUCTION

We currently live in a world ruled by large amounts of data. Organizations' success is highly determined by the way they foresee and assess changes occurring in the future. Predictive data analytics is the art of building and using models that create forecasts based on patterns extracted from historical data. So, it is a process of making projections about a specific event which the outcome is still unknown in the present. One of the main applications is price prediction (Kelleher, Namee, & D'Arcy, 2015). Price prediction can be applied in innumerous types of business, including the energy sector. Additionally, Big Data has created opportunities for development of new energy services and bears a promise of better energy management and conservation (Grolinger, L'Heureux, Capretz, & Seewald, 2016). Whenever prediction deals with time-series data, it can be designated as forecasting.

The electricity spot prices (ESP) represent the result of the market bidding prices outcome, in the electric wholesale market. Predicting these prices is an important and impactful task for market participants, like producers, consumers and retailers, since the principal objective for such players is to achieve the lowest cost in comparison with competitors. ESP play a huge role in energy market's decision making. It is important both for developing proper bidding strategies as well as for making conscient and sustainable investment decisions (Keynia & Heydari, 2019). Additionally, it impacts the decision of the technologies to use, for example, choosing between renewable energy generators or classic gas turbines. Furthermore, the topic of electricity prices forecasting is extremely relevant for both developed and developing countries. Developed countries search for their economic prospect's improvement. Electric energy efficiency is a crucial metric for that improvement. Electric energy efficiency can decrease the electricity prices thanks to the reduction of consumption, thus decreasing the need of having new expensive power generation and diminishing the pressure on energy resources. Therefore, ESP behavior is an important factor in their economy. Regarding developing economies, which have faced problems to take the populations out of poverty, the electricity sector restructuring has been fundamental for helping increase the levels of economic development (Ebrahimian, Barmayoon, Mohammadi, & Ghadimi, 2018).

ESP are represented by a time-series. A time-series investigation starts by careful examining recorded data plotted over time. This scrutiny regularly adopted indicates not only the method of analysis, but also the statistics that will be of use in summarizing the information in the data (Shumway & Stoffer, 2016). There are many different models to forecast ESP, and different parameters to select and measure. Essentially, forecasting models can be divided in two main groups: *Statistical* models and *Computational Intelligence* (CI) models. In the literature, statistical methods and CI have been used with success in numerous applications of time series forecasting. Also, hybrid models that bring together traditional statistical models and CI models have achieved relevant results when it comes of accuracy in different fields of application (Domingos, de Oliveira, & de Mattos Neto, 2019).

ESP forecast is a challenging task, since prices show high volatility and the electricity market has complex conditions (Qin et al., 2019). Several different factors can influence the prices, like overall supply, demand for power, tariff regulations, market conditions, the climate and weather conditions of the environment (sun, cloudy days, wind…), making electricity market prices increase or decrease over time. ESP forecasting is also a recent and broad topic. There are different approaches that can be adopted to perform an electricity price forecasting study, especially taking into account the forecast horizon, the data resolution, the historical data available and the market we are dealing with. Therefore, the techniques adopted in past studies covering this problem cannot be blindly applied in this dissertation. Our literature survey revealed a large gap regarding the number of studies in terms of a mid-term horizon forecast (several months to one-year time horizon), in contrast to short-term price forecasting. The studies that used Portuguese ESP data (Ferreira, Ramos, & Fernandes, 2019) are very sparse and in the case of mid-term scope using computational intelligence models, do not exist at all, as far as we know. One of the reasons is due to the fact that Portuguese electricity market total liberalization just occurred in 2007. Also, to the best of our knowledge, there is no research comparing different statistical and computational intelligence techniques in simultaneous in the scope of mid-term forecast in the Iberian market (OMIE), using Portuguese electricity spot price data.

Based on this brief introduction, the primary purpose of this dissertation is to implement and compare CI and statistical models to forecast the daily and monthly Portuguese ESP for the day-ahead OMIE market, with a one-year horizon period. This contributes to fill the mid-term Electricity spot prices forecasting research gap. In addition, the results of the predicted prices will serve ADENE – The Portuguese Agency for Energy (private non-profit association with a public interest), contributing to the consumers empowerment to reduce their energy bill, by predicting the energy prices fluctuations so that they are better informed when switching energy suppliers with the platform Poupa Energia – Save Energy (https://poupaenergia.pt/) through the supplier change logistics operator, OLMC (https://olmc.adene.pt/).

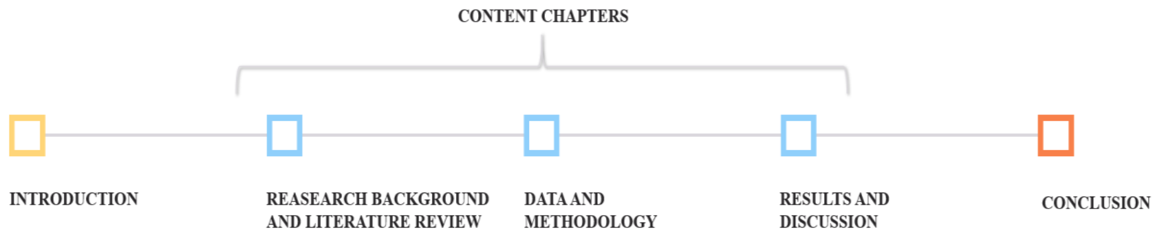
For the purpose in this study, we propose the following Research Questions (hereinafter: RQ). RQ1: What are the most used techniques and models in the literature, for a mid-term electricity price forecasting? RQ2: Which models have the best performance and the most accurate results, for a mid-term ESP forecasting, in the day-ahead OMIE market, using monthly and daily Portuguese data?

The following Research Hypotheses are proposed (hereinafter: RH). RH1: Computational Intelligent models show better performance if compared to Statistical models, when forecasting mid-term ESP in the day-ahead OMIE market. RH2: The forecast of mid-term ESP in the day-ahead OMIE market, using Computational Intelligent or Statistical models, has better performance when using monthly price data than daily price data. RH3: It is possible to improve the forecast of mid-term ESP in the day-ahead OMIE market, by selecting an ensemble of models, either trained with monthly or daily price data.

To identify and study the papers that implemented a mid-term electricity spot price forecast, we used the *PRISMA - Preferred Reporting Items for Systematic Reviews and Meta-Analyses method* (Moher, Liberati, Tetzlaff, & Altman, 2009) literature survey methodology, using Web of Science (webofknowledge.com) as the dataset and search engine to implement the developed queries. With this approach we aim at finding the relevant papers and identify the techniques and models used in them.

The development of the data exploration and the models' implementation to come up with the best electricity market clearing price forecast for a mid-term scope will mainly be based on the *CRISP-DM methodology*, Cross-Industry Standard Process for Data Mining (Wirth, 2000). As a data mining methodology, it proposes descriptions of the phases of a project and the tasks involved (Business Understanding, Data Understanding, Data Preparation, Modeling, Evaluation and Deployment). As a process model, CRISP-DM provides an overview of the data mining life cycle (IBM, 2016). All the data processing and analysis will be done in Python and R (programming languages). The selection of Python and R is justified due to their dynamic, intuitive, well suited to interactive development and prototyping characteristics, showcasing good and vast library options, appropriate for time series forecasting. Furthermore, since both programming languages were used in different courses in the candidate's master's degree, the candidate's proficiency with these languages is sufficient for the requirements of this thesis.

Regarding the ***Chapter Content Outline***, the thesis is divided in three main chapters, in addition to the introduction and conclusion chapters. The first main chapter consist in the *Research Background and Literature Review*, where an up-to-date description of the current state of European's electricity market and its participants, as well as an overview of the Iberian Electricity Market, are presented. Also, a description of the electricity spot price forecasting horizons, types, and models is done. Finally, a Systematic Review and Meta-analysis of the Literature is explained and the selected statistical and/or CI models to be used in this thesis are briefly described. The second main chapter consists in the *Data and Methodology*, where the data is explored, described and prepared and the models are implemented. The third main chapter consists in the Results and Discussion, where the findings are summarized and the model's results are compared. Finally, the conclusion of this master thesis is presented.



1 RESEARCH BACKGROUND AND LITERATURE REVIEW

This chapter serves as a presentation of the scientific foundation needed for this dissertation. This framework contributes to the understanding of important concepts and gives the reader the rationality and logic that lies behind this study. First, an overview of how the European electricity market is structured is presented, then a deep understanding on how the prices can be predicted, followed by a systematic literature review on mid-term electricity prices forecasting studies. This chapter sets the background for a better comprehension of the data and methodology chapter.

1.1 European Electricity Market

Electricity plays a fundamental part on modern societies like European ones, especially due to the fact of the fast growth and intensification of industrialization in the last two centuries. It gradually became a fundamental resource for many tasks of our everyday life. Therefore, it is crucial to have a system available in society that enables electricity's purchases and sales. This system is called electricity market. The main goal of the electricity market is then to provide reliable electricity at the lowest possible cost to consumers, be efficient, make the best possible use of the available resources, and enable long-run efficient investments (Cramton, 2017).

1.1.1 Physical Characteristics of Electricity

Electricity is a very singular commodity, having some specific and unique characteristics. First and foremost, before the advent of modern battery storage solutions (Ertugrul, 2017) and local community-based production, still in its early stages of adoption by consumers, electricity has been considered as economically uncapable of being stored in large quantities, having to be consumed at the same time that it is being produced. This condition makes electricity more similar to a service than to a good (Mäntysaari, 2015). Although, we cannot consider it as a proper service, so it is defined as a *tradable commodity* (Shah & Chatterjee, 2020). Additionally, electricity's consumption and production levels are highly volatile, they must be controlled and very well balanced. Electricity demand (also known as load) is measured in power units (megawatt) and reflects the sum of the quantity needed in a specific moment by the consumer plus the losses. The way electricity is transferred and commercialized is very specific, considering it must have a conduct material to transfer it and a grid and lines to transmit it for transportation from local production or import, and distribution to the consumer. Another peculiar characteristic is the fact electricity can have two different classifications as a tradable commodity. It can either be defined as the *flow* of electricity (average power) from one point to another in a specific moment and location, or it can be defined as the *accumulation* of the total energy in the grid in a specific moment and location (Mäntysaari, 2015). These are some of the main physical characteristics that heavily impact the electricity market structure and how it works. The efficiency of electricity market is a challenging task. Minor perturbations can cause huge repercussions in the electrical system.

1.1.2 European Electricity Market Evolution

In Europe, before the 90's, electricity was defined as a monopoly utility. This monopoly industry had vertically integrated structures for the electrical systems and was defined by the inexistence of competition, so big companies could decide and control the electricity's prices. Considering that those companies had the possession of the grid infrastructure too, there was no opportunity for new players to enter the market. Recently the worldwide electricity sector suffered an enormous change thanks to the trend of implementing a deregulated and liberalized market. This was valid for the European Union too, which decided to gradually remove and reduce some state regulations and restrictions, implementing a competitive electricity market, starting around 1996. This became a very heterogenous and impactful transformation, given the fact each country had its own electrical systems and infrastructures (Chicco, 2009). The goal was to achieve a single integrated internal European electricity market transversal to all EU states, which is now enforced. This redesign of the electricity sector, currently spreading globally, impacts how the exchange of electricity is made around the world. The exchange between each regional market is now easier, thanks to the implementation of the electric power exchange (EPX) and market coupling operator (MCO) (Mäntysaari, 2015). Furthermore, the competition between private players (producers and retailers) emerges, making the prices drop and the monopolies break. Consumers can now choose the supplier that is better suited for them. So, for the European case, the purpose of creating a single interconnected European market lies on three main reasons: give access to energy that is affordable, make electricity prices competitive, and create a more sustainable environment (Lam, Ilea, & Bovo, 2018). This results in an improved and more efficient European power system. It is important to note that a deregulated electricity market is not the same as a free interference of the state in the market.

1.1.3 Electricity Market Participants and Functions

According to the *EU Electricity Trade Law* (Mäntysaari, 2015) and the *Electricity Market Functions* (Energy Community, 2020), an overview of the main participants in the current electricity system (i.e. after the energy market liberalization) is presented below.

Producers: The ones who generate electricity in big power generation stations using different sources, i.e., power plants that transform primary energy (e.g., coal, gas, renewable energy sources such as wind, solar or biomass, nuclear power, etc.) into electric energy.

Large Consumers: End consumers that are big entities that might consume extreme quantities of electricity (high-voltage grid level) and, because of that characteristic, are interested in taking part in the electricity market, looking for a better price deal. Public institutions, large industrial or commercial firms are examples of large consumers.

Electricity Retailers: The ones that take care of buying electricity from the producers and selling electricity to the end consumer. They can be called as load-serving entities (LSE).

Small Consumers: The end consumers represent the domestic, residential households, or small businesses. Their role is fundamentally choosing a retailer and consume electricity. They might pay the local Distribution System Operator.

Prosumers: The ones that produce and consume electricity as well. A good example are the big companies that have their own power generator, or local household communities.

Transmission System Operators (TSO): The entity responsible for the high-voltage electricity transportation grid development, exploration, maintenance, and support. Plus, in charge to transport electricity from the power generators, by means of the transportation grid.

Distribution System Operators (DSO): Low- and medium-voltage grid operations, distributes electricity to the small consumers, via a dedicated network, called the distribution grid. Compared to the TSO, requires less monitoring but has more customers.

Market Regulator: Government entities that regulate a fair, clean and economically effective electricity trade, according to the rules and legislations, guaranteeing that the free market works in a fair and stable way.

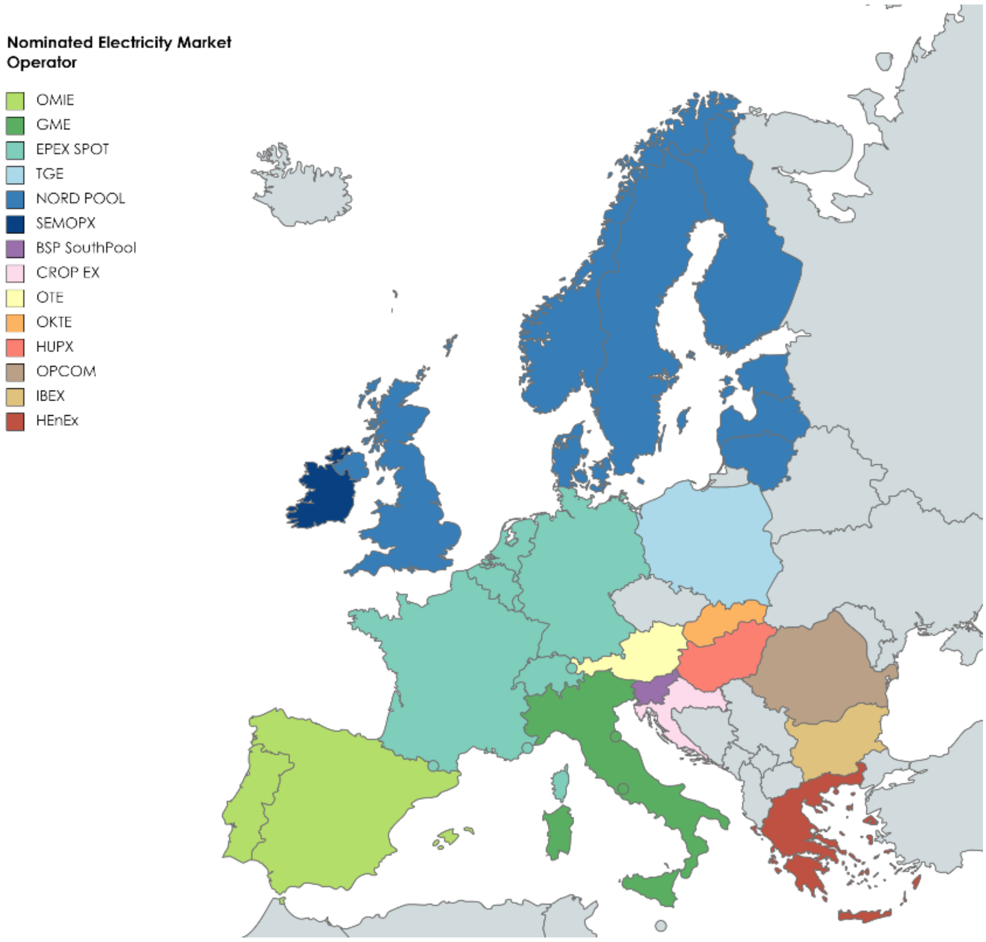
Market Operator: Entity that performs the “system” roles in the electricity market. Takes care of the means and conditions for an efficient electricity trade in the free market. Aggregates and matches all the bids of consumers and sellers, through a computer system.

Electric Power Exchange (EPX): An on-line platform mechanism where market participants can submit their demand or supply bids and proceed to their trades. It acts as the host for the markets, providing them with relevant information, settling the market clearing price (MCP) and addressing financial risks (Shah & Chatterjee, 2020). The EPX operates the spot market (Lam, Ilea, & Bovo, 2018). The main principles of power exchanges are liquidity, competition/open market, non-discriminatory treatment and anonymity and clearing/settlement. Currently, there are fifteen Power exchanges across the EU.

Nominated Electricity Market Operator (NEMO): A NEMO is a market operator (typically corresponds to the EPX) designed by the EU to operate and participate in the day-ahead or intraday coupling of neighboring markets. A NEMO must be defined per bidding zone. Bidding zones are a geographical area where network constraints are applied (NEMO Comitee, 2019). NEMOs act as a form of national or regional market operators. TSO and

NEMO must jointly cooperate in a European level. The existing NEMOs are colored in Figure 1.

Figure 1 Map of the European Nominated Electricity Market Operators



Source: Own Work.

Market Coupling Operator (MCO): Cooperation between NEMOs, a way of integrating neighboring markets into one coupled market. The main goals are to create an interconnected European Electricity market that links and unifies different electricity exchange systems. The MCO helps reduce price volatility across Europe and brings advantage to market participants, that do not have to acquire a transmission capacity right for a transaction across borders. Europe's Single Day-Ahead Coupling (SDAC) is responsible for integrating all day-ahead European markets, unifying them in one single day-ahead market that covers the entire EU. All the NEMOs must cooperate. The coupling is based on the Euphemia Algorithm (an-European Hybrid Electricity Market Integration Algorithm) (NEMO Comitee, 2019). On the other hand, the Single Intraday Coupling Market (SIDC) integrates all the continuous markets. To achieve this, a project called Price Coupling of Regions (PCR) was formed in 2009 by seven EPXs in Europe (EPEX SPOT, GME, Nord Pool, OMIE, OPCOM, OTE and TGE). The advantages of PCR are enhancing the liquidity of the market, reach an overall social welfare and implicit allocation management (Lam, Ilea, & Bovo, 2018).

1.1.4 Current Market Design

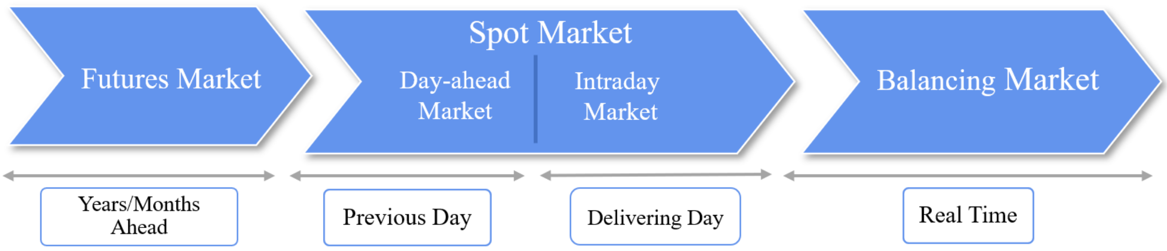
With the liberalization, companies stopped having the power of controlling simultaneously electricity's production, transportation, trade, supply, and the transmission and distribution networks. So, now we have a vast and complex electric sector that can be divided into two main Electricity Markets: The Wholesale Market and the Retail Market. The Wholesale Market is the foundation of a restructured electricity market, where producers compete to serve load (Cramton, 2017). It gives information about electricity prices. It is where retailers and large consumers bids take place, and the power producers trade big quantities of energy with the retail suppliers (to later serve their small consumers) and large business (Mäntysaari, 2015). On the other side, the Retail Market is where the small end consumer buys his electricity from the retailers (Mäntysaari, 2015). The Wholesale Market is the one responsible for the equilibrium of the whole system. The wholesale prices are variable and fluctuate over time, while the retail prices are normally presented at a fix rate. Typically, the wholesale prices are lower when compared to the retail prices. In this study we will be focusing on the Wholesale Market. The electricity market design after liberalization can be summarized in Figure 2.

Figure 2 Schematic diagram of the current electric market structure

Source: Own Work.

As seen, the key purpose of the wholesale market is to ensure that the electricity levels are well controlled and in balance. For that to happen, the wholesale market is built in a way that the participants have enough room to commit “errors”, make changes and do some adjustments in their planning. To achieve that, the Wholesale Market is organized according to how far in advance the electricity is traded. Figure 3 shows the different types of markets encompassing the wholesale market. We have a Futures Market; Spot Market (arranged in a Day-ahead and an Intraday Market); and finally, a Balancing Market (Mäntysaari, 2015).

Figure 3 Markets encompassing the wholesale market



Source: Own Work.

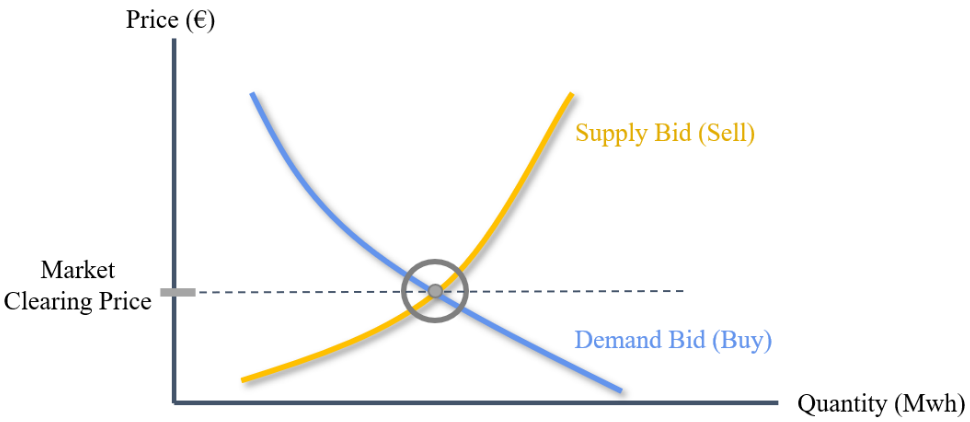
1.1.4.1 Futures Market

Futures Market is where long-term contracts take place (months to years ahead). Forward contracts are the most common ones. They consist in establishing in advance the fixed quantity and price of electricity for a settle moment in time. Other examples of futures markets are base-load contracts, total supply contracts, peak load contracts or reserves (Mäntysaari, 2015). The main advantage of this market is the opportunity for the participants to better manage their energy security risks (Cramton, 2017).

1.1.4.2 Day-Ahead Market

The Day-Ahead Market is where the market participants sell and buy electricity, in a short-term period, the day before of the transaction happening. The day-ahead market has the objective of dealing with the participant’s sale and purchase bids, that must be submitted in the day *d-1*, so the prices of the following day *d* will be established. Therefore, every day of the year at 12:00 CET, a session is taken to establish the next day price and volume for each specific hour (OMEL, 2019). So, for each hour of the day, a unique price is defined. The electricity spot price, more specifically the day-ahead price, is determined by the Market clearing price (MCP). MCP is what is defined by the meeting point between the supply and demand bid curve, i.e., where the equilibrium lies. Since suppliers are the ones that want to sell more at a higher price and buyers the ones that want to have more quantities at a lower price, the bid curves normally reflect a shape similar to the ones seen in Figure 4. The bids are made by 1 up to 24 blocks corresponding to each hour of the following day (or 23 or 25 on the days that the change of hour occurs according to the official calendar), setting up the volume and prices offered of each block. The bid submissions are done through the NEMO of the respective countries.

Figure 4 Market clearing price by aggregation of supply and demand



Source: Own Work.

Matching Procedure on The Day-Ahead Market

The Market Operator will deal with matching the bids via the Euphemia Algorithm (Pan-European Hybrid Electricity Market Integration Algorithm). This algorithm was created to help with the coupling of the day-ahead electricity markets in the PCR. It is computed at a bidding zone level. It was applied for the first time in 2014. The algorithm consists on the optimization of the Social Welfare (*SW*) and returns the MCP, volumes and net position correspondent to each specific bidding zone (NEMO Comitee, 2019). The *SW* consists on the total sum for the planned hour horizon of the Surplus of Submitted Supply (*SS*), plus the Surplus of Demand Bids (SD), plus congestion rent across regions (ST) (Sleisz & Raisz, 2017). See equation (1). The *SS* corresponds to the gain from the sale bids and is the difference between the marginal price received, i.e the bid's actual income (INC_k for bid K) and the bare minimum quantity the seller is willing to receive ($IncAsBid_K$). See equation (2). The *SD* corresponds to the gain from purchase bids, and is the difference between the maximum price the consumer is willing to pay ($ExpAsBid_K$) and the actual price paid, according to the MCP (EXP_k for bid k). See equation(3). The objective is achieving the SW's Maximum value.

$$SW = SS + SD + ST \tag{1}$$

$$SS = \sum_{k \in Sup} (INC_k - IncAsBid_k) \tag{2}$$

$$SD = \sum_{k \in Dem} (ExpAsBid_k - EXP_k) \tag{3}$$

1.1.4.3 Intraday Market

Intraday market is a supplementary platform that supports the day-ahead market. It helps adjust and balance the electricity prices traded in the daily market, after the day-ahead market. This market lets buyers and sellers react to unforeseen changes and correct their position before the physical delivery takes place, balancing the electricity levels of supply and demand. The adjustments are done after the results are sent to the system operators. Participants can negotiate the contracts with a limit of 1h before the delivery (Shah & Chatterjee, 2020). These markets are managed in different sessions throughout the day. The market is subdivided in two types: The Intraday Auction Market and The Intraday Continuous Market (also known as Single Intraday Market).

1.1.4.4 Balancing Market

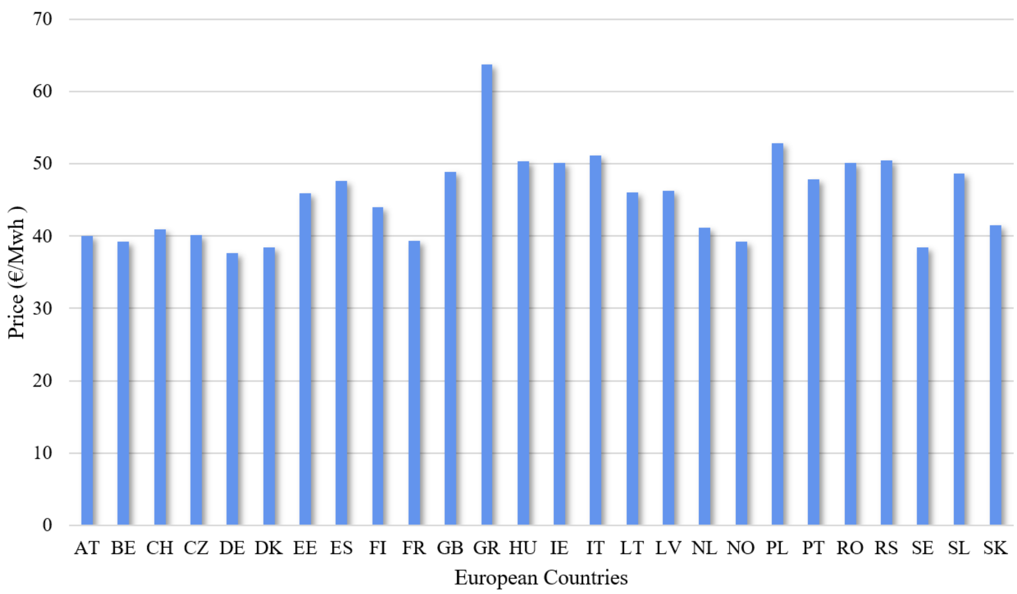
The system operator must take care of the balance on the grid, planning in advance and doing some estimations about the future values of production and load levels. However, the real values can only be acceded in real time. So, the system operator must fill the gaps and counterbalance the energy levels and reserve in real time while they keep changing. The cost of maintaining the equilibrium of the grid is divided by the market entities that created the imbalance, i.e. the entities that produced or consumed higher or lower quantities than the already established and agreed upon ones, in the previous markets (Mäntysaari, 2015).

1.1.5 Iberian Electricity Market

The Iberian Market of Electricity, designated by MIBEL, is constituted by Spain and Portugal's electricity markets and started to operate in the 1st of July 2007 (MIBEL Regulatory Council, 2009). It became part of the European Market in 2014 for the day-ahead horizon and in 2018 for the intraday horizon. It is managed by OMI (Iberian Market Operator – Operador de Mercado Ibérico). MIBEL provides the Iberian consumers a free market for any retailer, large consumer or producer in those two countries. OMI is divided into two companies, OMIP SGPS (located in Madrid) and OMEL (located in Portugal). Each one of them owns 50% of OMIE and 50% of OMIP. OMIE is the Iberian Peninsula's NEMO. Not only it is responsible for the electricity's day-ahead market and intraday market, but also establishes the connection with the other NEMOs of Europe. OMIP is mostly committed to the forward market. It deals mainly with derivatives products, like options, swaps, forwards, and futures. OMIE manages and puts in order the producers and suppliers' bids from the lowest to the highest. These bids are prices that range from 0 €/Mwh to 180.30 €/Mwh. These price limits are designated as instrumental prices (MIBEL Regulatory Council, 2009). Figure

5 shows a comparison between the average of the day-ahead electricity spot prices in 26 European countries in the year 2019. The data was retrieved from (ENTSO-E Transparency Platform, 2020).

Figure 5 Average of the day-ahead ESP in 2019 in Europe



Source: Own Work, based on (ENTSO-E Transparency Platform, 2020) data.

Focusing now only in Portugal, based on the information available by *ADENE - Agency for Energy* in the portal “Portugal energy” (https://www.portugalenergia.pt/agentes) and based in the latest available report “Statistic Report September 2018” (OLMC, 2018) about the retailers in the electricity liberalized market, we can name the main participants in the Portuguese electric sector below.

Producers *(ordinary regime, not covered by juridical legislations):* EDP, Turbogás, Tejo Energia, ELECGAS

Transmission System Operators (TSO): REN (Redes Energéticas Nacionais).

Distribution System Operators (DSO): The main one is EDP Distribuição, followed by EDA (Eletricidade dos Açores) and EEM (Empresa de Electricidade da Madeira). There are 11 more small distributors.

Retailers: EDP Comercial (81,07% share) Endesa Energia, Sucursal Portugal (5,46% share) Galp Power, S.A. (5,13% share) Iberdrola Generación - E.S.P.U. (4,31% share) and 37 more (total share of 4,03%).

1.2 Electricity Spot Price Forecasting

Electricity spot prices are represented by a succession of values in a specific period of time, also known as a time-series. Forecasting utilizing time-series analysis involves the use of some substantial model to predict forthcoming outcomes based on known previous results. Therefore, electricity spot price forecasting (ESPF) consists in predicting the spot prices in the wholesale market on a certain period in the future, based on the prices observed in the past.

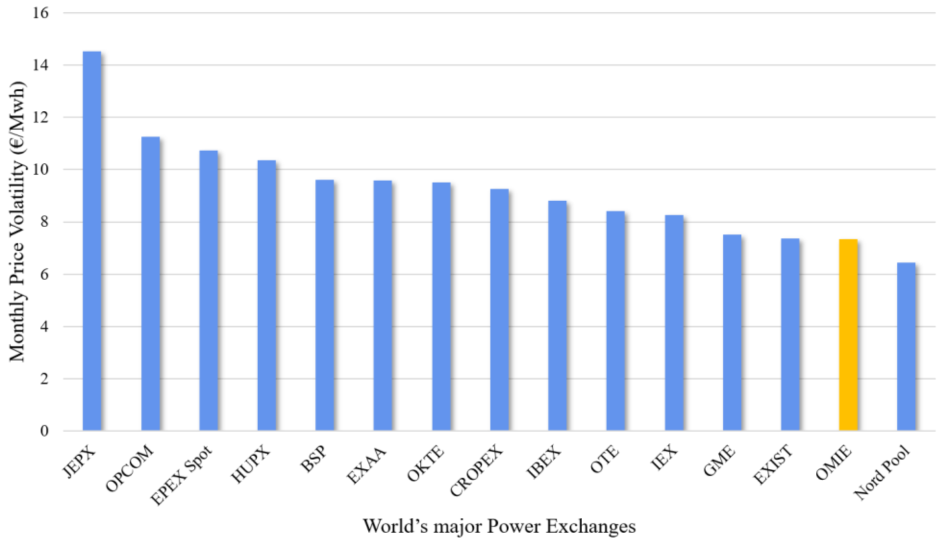
In some studies related to electricity forecasting, some authors have pointed that electricity spot prices can be influenced by multiple factors such as daily activities, business intensity and weather, making them challenging to predict (Peng, Liu, Liu, & Wang, 2018). This makes ESPF a difficult task, especially because some of the external factors that might influence prices are also hard to predict. Natural gas prices and meteorological variables, like temperature, sunshine and precipitation are, to some extent, considered easier to manage and predict in a short-term horizon. The same does not apply to variables such as MCP bidding strategies, levels of production, levels of consumption, electricity demand and supply, spinning reserve market price, transmission loss, business strategies and even unethical business behaviors.

1.2.1 Electricity Spot Price Volatility

Electricity spot prices are characterized for their fluctuations, nonlinear behavior, randomness and non-stationarity (Peng, Liu, Liu, & Wang, 2018). Electricity spot prices of the day-ahead market fluctuates a lot, reflecting a high volatility. The occurrence of unexpectant pick prices is very common, being an issue to electricity contracts. This volatility plays a huge role in the decision making of the electricity market participants of any EPX, since the risk of trading is directly correlated to that volatile behavior. If the volatility increases, the uncertainty and risk are higher (Shah & Chatterjee, 2020).

The spot price volatility is a consequence of electricity distinct characteristics. Rapid fluctuations in the prices can occur especially due to the imbalance of consumption and production levels. When the observed consumption levels are below the expected ones, the prices tend to decrease. In the other hand, if the production levels are below the expected, the prices rise (Mäntysaari, 2015). Moreover, besides demand and offer instability, the volatility is swayed by the limitations of transmission capacity and the expenses of production. High expenses can cause a disinterest in electricity production investors. Plus, the prices are influenced by the country's primary energy source that generates electricity, the external dependency, and the financial risk (Biçen, 2019). Hence, the market prices levels and volatility differ from country to country. In Figure 6 we represent the day-ahead spot prices volatility of the world's major *Electric Power Exchange*s. The data is retrieved from (Shah & Chatterjee, 2020). OMIE is below average.

Figure 6 Volatility of the day-ahead ESP by some of the world's major EPX



Source: Own Work, based on (Shah & Chatterjee, 2020) data.

1.2.2 Forecasting Horizons

Forecasting the ESP is a vast research topic, that can be narrowed down in different scopes. Clearly understanding and describing the horizon of the forecast is extremely relevant, since the techniques and models to be selected will be different according to each horizon. Similar models and tools can be applied in all of the different horizons, although the way they are applied requires distinct approaches and care (Weron, 2014). Unfortunately, there is not an official and consensual established definition regarding the electricity prices forecast horizon's scope. So, the horizon's definition for this study will be defined based on the definition adopted by the most referenced, relevant, and prominent authors in this research area, like Rick Steinert from *Universitat Viadrina*, Florian Ziel from *Universitat Duisburg-Essen*, and Rafal Weron from *Wroclaw University*.

1.2.2.1 Short-term Horizon

When the forecast goes from minutes up to few days, we are in the presence of a short-term horizon. Weather variables are relevant and impactful in the short-term ESPF. It is important for day-to-day market operations and system stability (Weron, 2014). Demand and offer benefits from this forecast by establishing more accurate bids and efficient trades.

1.2.2.2 Mid-term Horizon

A mid-term (or medium-term) forecast covers forecasts from weeks up to one year. Weather variables do not have huge relevance in this horizon, since a reliable, useful and accurate forecast of the weather can only go up to 3 to 10 days into the future (Bauer, Thorpe, & Brunet, 2015). Also, political and technological uncertainties do not cause a major impact, due to the fact that those factors do not have drastic changes during a period of just some weeks or months. On the other hand, seasonal consumption impacts this horizon (e.g. consumption reduction in holiday period), as well as power generation variables (Maciejowska & Weron, 2016). The importance of this forecast horizon mostly lies in the need for a mid-term plan, including adjustment of schedules, allocation of resources, establishing bilateral contracts, assessing derivatives and managing risk (Weron & Ziel, 2018).

1.2.2.3 Long-term Horizon

A long-term horizon consists in a forecast period superior to one year. Political, cultural, technological, social, regulatory, and economic uncertainties heavily impact this horizon. All those variables are hard to predict, making a long-term forecast a more difficult and challenging task. It is relevant for investment planning and policy making (Ziel & Steinert, 2018).

It is important to note that mid-term and long-term forecasts are more complex tasks than short-term, since they deal with a longer forecast period and a good accuracy of the prediction is harder to achieve. Short-term forecast is a more comprehended task nowadays, even though it is still far from being totally understood. Also, it is harder to have good results in mid- and long-term horizons compared to short-term, given that historical data is still somehow limited. The available data is still recent and not so vast, given that electricity liberalization is a recent measure.

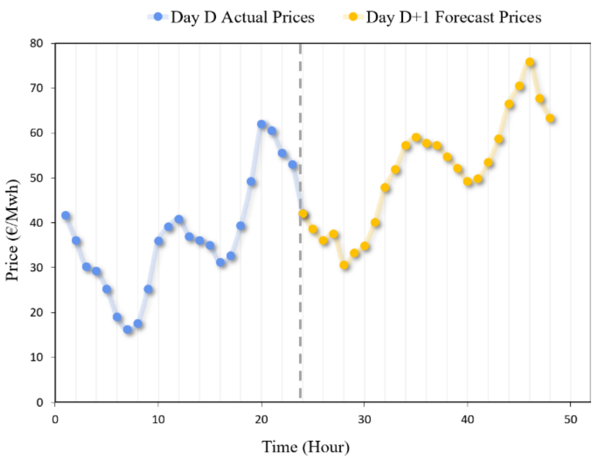
1.2.3 Forecasting Types

The electricity prices can be forecasted according to three different main forecast types: deterministic, probabilistic and ensemble. The majority of ESPF papers are focused in deterministic forecast, although thanks to the Global Energy Forecasting Competition – GEFCom2014, the number of papers related to probabilistic ESPF has increased (Hong, Pinson, Fan, Zareipour, Troccoli, & Hyndman, 2016).Therefore, from 2015 to 2018 15% of ESPF studies were based on interval and distribution predictions. Literature that covers ensemble forecasts is still very scarce (Weron & Ziel, 2018).

1.2.3.1 Deterministic

The deterministic or point forecast approach simply consists in predicting a single desired value of the price, given some conditional information, for each time instance. The nomenclature related to deterministic forecast is as follows: $P_{d.h}$ is the electricity price, where d represents the day and h the periods ($h = 1, 2, \ldots, H$). Consequently, the expected value of the price is represented by $E(P_{d.h})$ and the point forecast by $\hat{P}_{d.h}$. Figure 7 serves as an example of point forecast of the day-ahead D+1. The downside of the deterministic forecast is that it does not cover uncertainties, in contrast to probabilistic forecast.

Figure 7 Illustration of a deterministic forecast

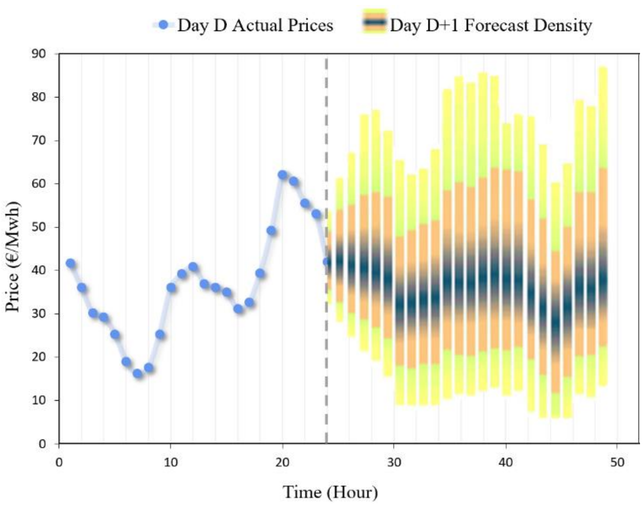


Source: Own Work.

1.2.3.2 Probabilistic

Like deterministic forecast, probabilistic forecast is concern in predict events, although this one also gives information about the probability of occurrence and magnitude of a specific event. With this type of forecast, the probabilities of the possible outcome of a random variable can be estimated. So, probabilistic forecast tries to quantify uncertainty and provide information about risk exposure. It can be implemented from two different approaches. The first one calculates the error distribution of the point forecast. The second one considers the probability density function of the prices (Weron & Ziel, 2018). Figure 8 serves as an example of the density forecast of the day-ahead D+1.

Figure 8 Illustration of a probabilistic forecast

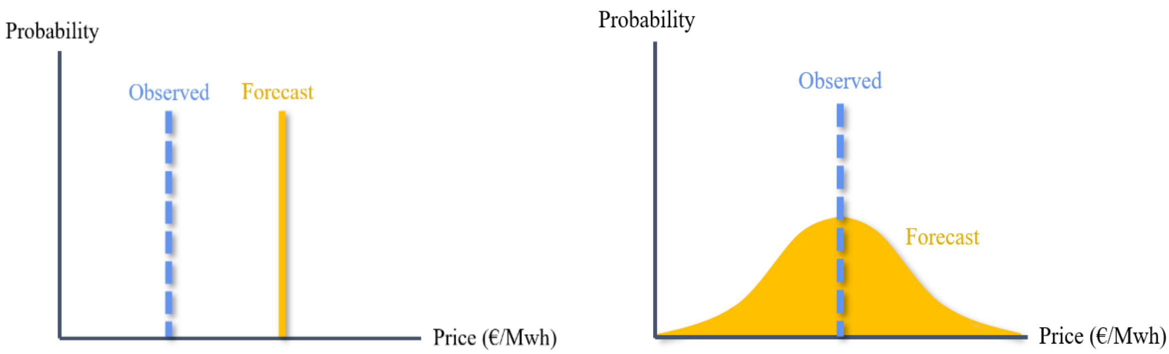


Source: Own Work.

As stated before, $\hat{P}_{d,h}$ is considered the future price. So, in this case, we have $P_{d,h} = \hat{P}_{d,h} + \varepsilon_{d,h}$, where $\varepsilon_{d,h}$ denotates the error. At this point, $\hat{P}_{d,h}$ is still a single value forecasted. We can adjust this definition into a probabilistic forecast by considering the following: $F_P(x) = F_\varepsilon(x - \hat{P}_{d,h})$, where F_P is the distribution of prices and F_ε the distribution of errors.

To summarize and have a better comprehension of the deterministic and probabilistic forecast, Figure 9 depicts a comparison between the two. While deterministic forecast implies 100% probability of occurrence, probabilistic forecast has many distributions representing different kinds of uncertainties.

Figure 9 Deterministic forecast VS Probabilistic Forecast.



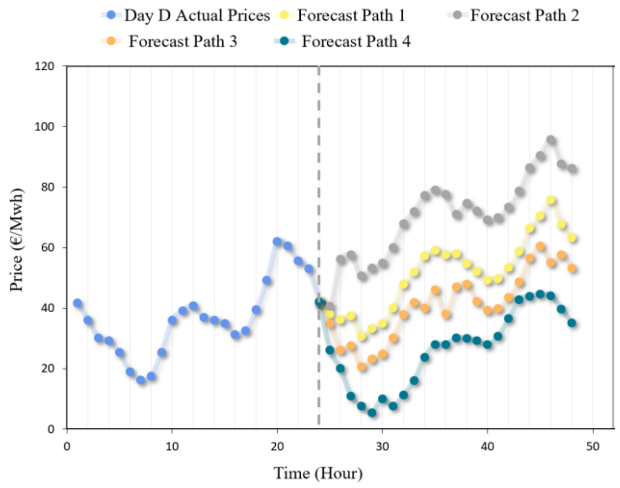
Source: Own Work.

1.2.3.3 Ensemble

Ensemble forecast is a Monte Carlo instance integration and it is also designated by *simultaneous prediction intervals*, *prediction bonds* or *prediction scenarios* (Weron & Ziel, 2018).

It consists in producing not one forecast but an ensemble of forecasts, grouping different scenarios of how the prices will behave in the future, starting from slightly distinct initial conditions. Each path is originated by a collection of single points integrated in space phases of specific times in the future, representing a statistical distribution of forecast uncertainty (Wilks & Vannitsem, 2018). These points are randomly generated, according to the given initial probability density. For each point (i.e. each ensemble sample), deterministic trajectories are calculated (Wilks, 2019). See Figure 10 for an example of this type of forecast.

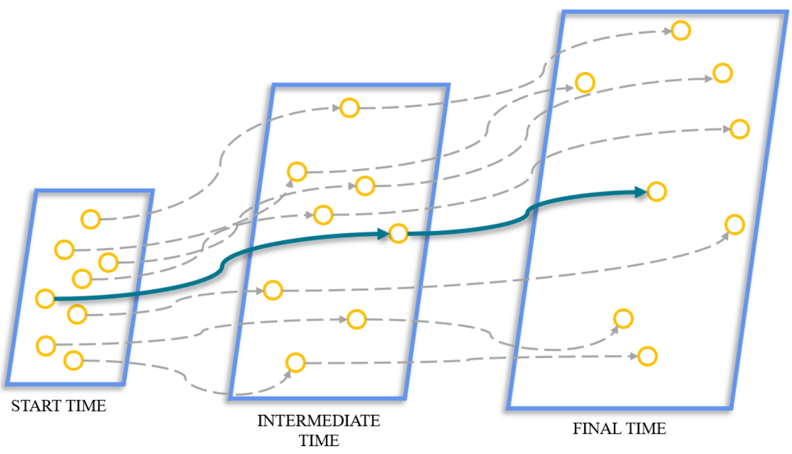
Figure 10 Illustration of an ensemble forecast



Source: Own Work.

For a better comprehension, an illustration of some concepts in ensemble forecasting are plotted in terms of a three-dimensional phase space in Figure 11. In this figure, we have a representation of the ensemble forecast, where the rectangles are a representation of each space phase plotted in a specific time. The first rectangle is the originated phase of which the first prices are originated, by the probability distribution. The small yellow circles represent the forecast price's points, and the grey dashed lines are the possible path of each point. The bold dark line provides the best evolution trajectory of the initial price values. In the initial time, the ensemble points are more similar, being close to each other. In the future time, with the advance of the forecasts, we can observe a more disperse placing of the points in each space phase, adopting qualitatively different flows. Each of the lines (ensemble members) are possibilities of the price's values, but it is difficult to know in advance which one will be closer to the real prices.

Figure 11 Schematic representation of an ensemble forecast



Source: Own Work.

The way the ensemble members are spread reflects the confidence of the price prediction. If they are closer and narrow together the prices will have a more accurate and restricted range possibility of values. If they reflect a sparser behavior, the possible outcome values of the prices will increase, making it a less accurate forecast. As time advances, an accurate forecast is harder to achieve.

1.2.4 Forecasting Models

In terms of the models the two groups most used to forecast electricity prices are: *Statistical* models and *Computational Intelligence* models. Nevertheless, Weron & Ziel (2018) state that three other models were also employed in some studies: *Reduced-form* models, *Fundamental* models and *Multi-agent* models. Furthermore, the literature on this topic provides several hybrid solutions. Below, the two universally most used groups of models are briefly introduced and described.

1.2.4.1 Statistical models

Statistical models, also designated as econometric models, represent the technical analysis of ESPF. They are applications of direct traditional statistics techniques (Weron, 2014). Normally this type of models consists of a mathematical and weighted combination of past price's values with variables that impact the prices. Autoregressive is an important concept in statistical models, it describes the dependence between the forecasted prices with the past prices (Weron & Ziel, 2018). Statistical models are mostly based on regression models. The downside of statistical models is the challenge of dealing with nonlinear events, being limited when it comes to model nonlinear behaviors (Weron & Ziel, 2018).

1.2.4.2 Computational Intelligence models

Computational Intelligence (CI) models are also designated as *Artificial Intelligence or Machine Learning* models in the literature. This type of models can solve nonlinear problems, with the help of computational intensive tools, that linear statistical models cannot solve effectively. They are based on said "intelligent algorithms" that can be influenced by biological processes, using approaches that are a combination of learning, evolution and fuzziness elements (Weron, 2014). A good characterization of this area is hard to define, since some authors classify specific models under CI, while others consider that those same models are statistical models (Duch, 2007). "*Computational Intelligence is a branch of computer science studying problems for which there are no effective computational algorithms*" (Duch, 2007). Computational Intelligence can be seen as applied statistics. The distinction between them is often blurry.

1.3 Systematic Review and Meta-analysis of the Literature

We currently live in a world ruled by constant technological innovations (e.g., internet of things; high performance and cloud computing; virtual and augmented reality; artificial intelligence; natural and multimodal human-computer interaction; open-source algorithms; etc.). Alongside, our world is “flooded” with large amounts of data that increase daily, and that is, in most cases, available, with easy, rapid, and instant access. This new big data era is characterized by the 5 Vs: Volume: scale of data; Variety: different forms of data; Velocity: streaming of data; Veracity: uncertainty of data; and Value of data (Ishwarappa & Anuradha, 2015).

The advantages and advances of this new era, aligned with the rising progress of renewable energy sources and the liberalization of different and many electricity markets around the world, has impacted the electricity price forecasting research, bringing a new dimension, depth and opportunities with it. All of this resulted in the emergence of new papers regarding this topic. Weron (2014), stated that before 2000, there was a big gap in the literature, with a lack of papers covering this subject. However, that changed, and, from that point forward, the number increased through the recent years. Articles addressing electricity price forecasting came from different research areas, such as: Energy, Engineering, Computer Science, Mathematics, Business, Management and Economics. The majority of them focused on a short-term forecast. There is still a considerable gap when it comes to mid- and long- term forecast in the literature, and the ones tackling this issue have problems predicting accurate values of prices, given the fact they lack realistic price’s time series data (Ziel & Steinert, 2018).

According to Ziel & Steinert (2018), for mid- and long- term approaches, from the year 2000 till 2017, the most common models used in the literature are SVM - Support Vector Machine (CI approach) and Linear Regression (Statistical approach). Most of them include the added support of other models, showing variety and diversity in the proposed approaches and in the employed quality measures. So, for our study, it was necessary to search and look for the most recent used techniques in this specific horizon scope (mid-term). Papers that have a long-term scope were also included on this search, since some authors consider 12 months as a long-term horizon.

In order to summarize the studies related to the forecast of electricity spot prices and find the most used mid-term techniques in the literature, we adopted and implemented a systematic review methodology, following the guidelines of the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA methodology) (Moher, Liberati, Tetzlaff, & Altman, 2009). PRISMA consists in the inclusion and exclusion criteria of records and is organized in the following way:

1. Collection and identification of the relevant manuscripts.

2. Screening of the titles and abstracts.

3. Full text screening.

4. Final papers to be analyzed in detail.

In order to identify and collect the papers related to this study, we used the scientific paper repository *Web of Science*, which is considered to be the gold standard for research discovery and analytics (*wokinfo.com*). It is well-known, intuitive and has a vast library of scientific content. The queries used in the search engine to identify the relevant studies for our survey are presented in Figure 12. These queries are words joined by Boolean logical operators ("AND" & "OR") that are used to search and find article's titles, abstracts and keywords that contain the same words presented in those queries. Four queries were applied. The first query corresponds to articles related to electricity prices. The second one is related to forecast. The third one is focused on the horizon. The last one logically merges those three queries with the conjunction "AND". This final one is the query that gives us the ultimate final list of records. This query was applied in April 2019. In order to have a relevant and up to date result, the articles were filtered from the last five years (2015 to 2019).

Figure 12 Queries used in the Web of Science Repository

QUERY #1

TS= ("Energy Price" OR "Energy Prices" OR "Energy Pricing" OR "Energy Tariffs" OR "Energy Market" OR "Energy spot market" OR "Energy day-ahead price" OR "Energy day-ahead market" OR "Electricity Price" OR "Electricity Prices" OR "Electricity Pricing" OR "Electricity Tariffs" OR "Electricity Market" OR "Electricity spot market" OR "Electricity day-ahead price" OR "Electricity day-ahead market" OR "Natural Gas Price" OR "Natural Gas Prices" OR "Natural Gas Pricing" OR "Natural Gas Tariffs" OR "Natural Gas Market" OE "Natural Gas spot market" OR "Gas Price" OR "Gas Prices" OR "Gas Pricing" OR "Gas Tariffs" OR "Gas Market" OR "Gas spot market")

QUERY #2

TS= ("Prediction" OR "Predictions" OR "Predicting" OR "Predictive" OR "Predict" OR "Predictability" OR "Forecast" OR "Forecasts" OR "Forecasting" OR "Time Series" OR "Artificial Intelligence" OR "Machine Learning" OR "Probabilistic")

QUERY #3

TS= ("mid-term" OR "mid term" OR "medium-term" OR "medium term" OR "middle-term" OR "mid" OR "medium" OR "middle" OR "long-term" OR "long term" OR "long" OR "weeks-ahead" OR "weeks ahead" OR "months-ahead" OR "months ahead" OR "monthly horizon" OR "1 year" OR "one year" OR "twelve months" OR "12 months")

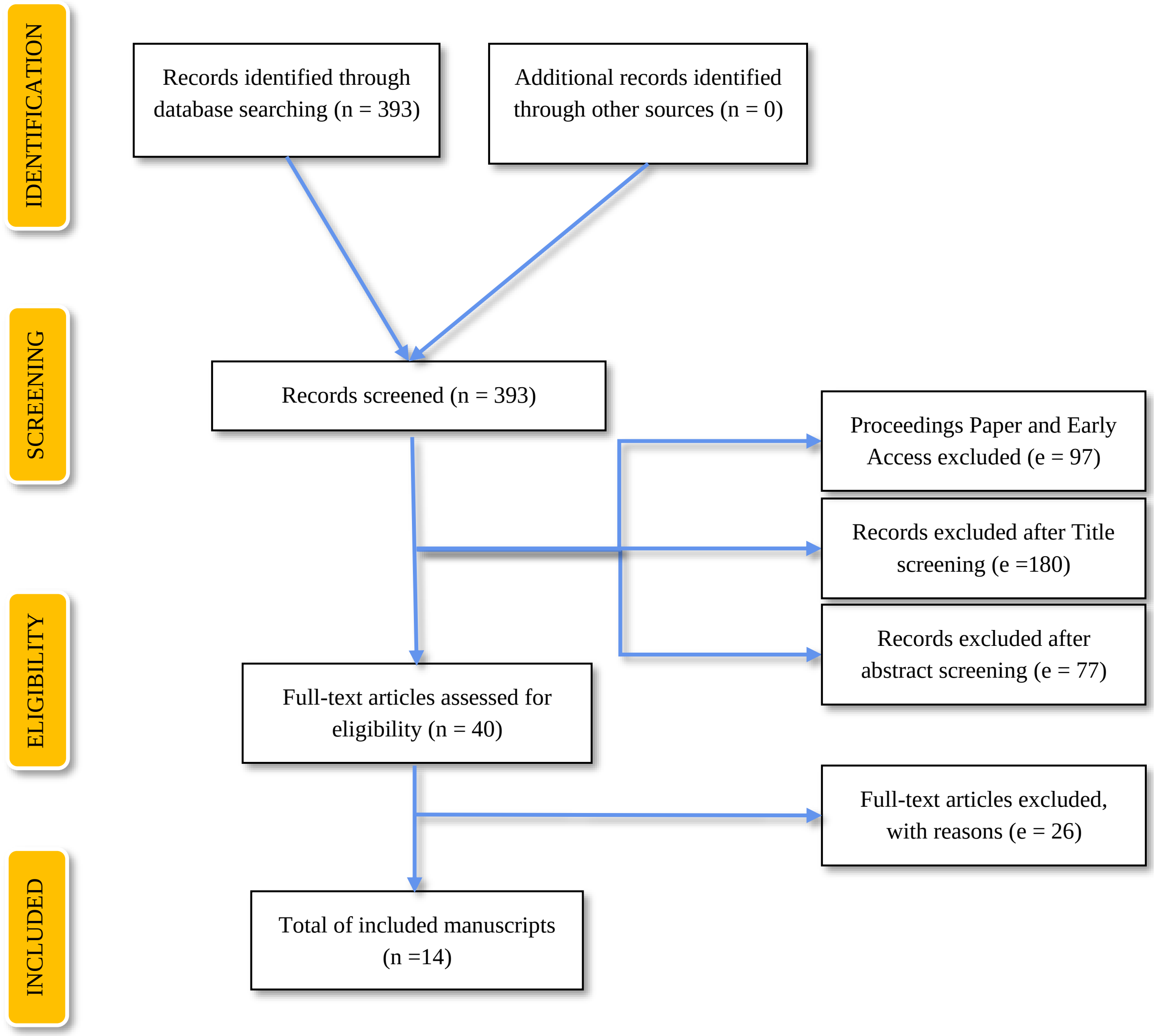
QUERY #4 (Final Query)

#3 AND #2 AND #1

Source: Own Work.

In Figure 13 we present the PRISMA diagram with the number of papers resulted after the application of each phase. In the initial phase (identification), after applying the queries, we filter the timespan, remove possible duplicates, and set the language preference to English, which results in retrieving a total of 393 papers. After that, exclusion criteria was performed along the phases, based on principles such as, papers not having an abstract, being classified as proceedings or early access papers or were not relevant for this study in particular.

Figure 13 PRISMA Diagram



Source: Own Work.

From the 393 records we end up with 295, after excluding 97 that were classified as Proceeding or Early Access papers. Next, we excluded 180 papers, from those 295, ending up with 115, excluding papers with titles that were not related to the aim of this study. From those 115, we end up with 40, after reading all the 115 paper's abstract and consequently removing 77 since they were out of scope. After that, we assessed such 40 papers for eligibility with full-text reading, removing 26 since they were out of the scope of our research, and ending up with a final count of 14. These 14 included articles are summarized in Table 1, ordered by publication date.

Table 1 Summary of mid-term ESPF papers, for the timespan 2015 – 2019

No.	Reference	Training Data Duration	Data Resolution	Data dimensions	Forecast Period	Compared Models *(Model that outperformed in bold)*	Accuracy Measures	Market
S1	(Steinert & Ziel, 2019)	1 year	Hourly (1h time interval)	Spot prices Future Phelix data Week Index	1 month	**AR24-X** AR24(p) AR-HoW(p) VAR-X(p)	- MAE - MMAE	Germany and Austria (EPEX)
S2	(Ferreira, Ramos, & Fernandes, 2019)	6 years (1 Jan 2010 – 31 Dec 2015)	Monthly (Average price of each month)	Spot Price s Electricity Consumption per capita Heating Degree Days Cooling Degree Days Industrial Production Index Hydroelectric Productivity Index Europe Brent Spot Price FOB Crude Oil Imports per capita Renewable Special Regime Production per capita Electricity Import-Export Balance per capita	1 year	**MLRM**	- MAPE	Spain and Portugal (OMIE)
S3	(Mujeeb, Javaid, Ilahi, Wadud, Ishmanov, & Afzal, 2019)	12.5 years (1 January 2006 – 31 March 2018)	Hourly (1h time interval)	Spot prices	1 month	**DLSTM** ELM NARX WT+SAPSO+KELM	- MAE - NRMSE	New York City (NYISO) and New England (ISO NE)
S4	(Razak, Ibrahim, Abidin, Siah, Abidin, & Rahman, 2019)	1 year	Monthly (Average price of each month)	Spot prices Month Index	1 month	**LSSVM+BFOA** SVM RBF-NN WNN MA SVM/SVM SVM/RBF-NN RBF-NN/RBF-NN RBF-NN/SVM Navigant Co.	- MAPE - MAE	Ontario, Canada
S5	(Windler, Busse, & Rieck, 2019)	5.7 years (1 January 2011 – 17 September 2016)	Hourly (1h time interval)	Spot prices Hour Day of the week Week Month Year	1 month	**DFNN** WNN TBATS	-RMSE -MAPE	Germany and Austria (EPEX)
S6	(Ziel & Steinert, 2018)	2.5 years (1 Nov 2012 – 19 April 2015)	Hourly (1h time interval)	Spot prices Temperature Generation power data (wind, solar, nuclear, lignite, coal, natural gas, hydro) Electricity load Auction data Dummy variables (for day/week/season/holiday)	3 years.	**X-Model (LASSO and Bootstrap)** AR-HoW	- ECP	Germany and Austria (EPEX)
S7	(Bello, Bunn, Reneses, & Munoz, 2017)	1.5 years (1 April 2013 – 30 June 2014)	Hourly (1h time interval)	Spot prices Load Wind power generation Hydro power generation Imports/exports Natural gas prices Coal prices CO2 prices Power plant costs	1 to 2 months	**Quadratic equilibrium model + Quantile regression** GARCH CAVIiaR	- WS - PBS	Spain (OMIE)
S8	(Cheng, Luo, Miao, & Wu, 2016)	1 year (April 2015 – March 2016)	Monthly (Average price of each month)	Electricity load Electricity export Number of generation companies Production (hydro, thermal, wind and solar)	1 month	**Grey prediction GM(0,N)** MLR Traditional GM(0,N) ANN	- MAE - MSE - MAPE	China Yunnan New
S9	(Bello, Reneses, & Muñoz, 2016)	3 years (1 January 2009 – 3O November 2011)	Hourly (1h time interval)	Spot prices Hydro and wind Electricity load Power plant costs Natural gas prices CO2 prices Coal prices Weekdays Holidays	3 months	**Logistic regression + Quadratic equilibrium model** Decision tree MLP	- BS - ECP	Spain (OMIE)

(table continues)

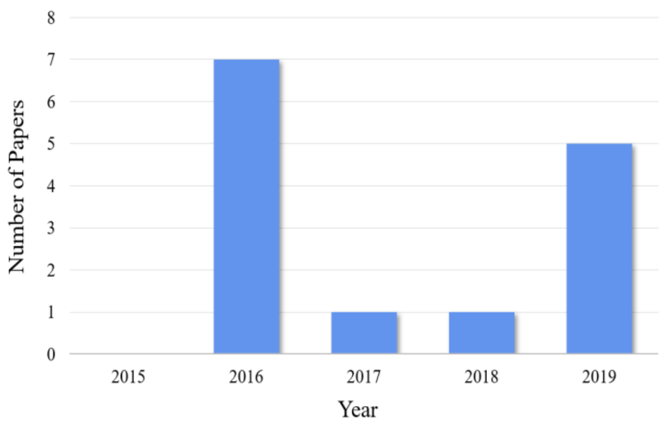
(continued)

No.	Reference	Training Data Duration	Data Resolution	Data dimensions	Forecast Period	Compared Models *(Model that outperformed in bold)*	Accuracy Measures	Market
S10	(Bello, Bunn, Reneses, & Muñoz, 2016)	7 months (1 May 2013 – 30 November 2013	Hourly (1h time interval)	Spot prices Wind hydro, import and export load fuel prices, power plant costs	6 months	**GAMLSS** Quadratic equilibrium Quantile regression Spatial interpolation Cointegration	- ECP	Spain (OMIE)
S11	(Ortiz, Ukar, Azevedo, & Múgica, 2016)	8 years (1 January 2004 – 31 December 2011)	Monthly (Average price of each month)	Spot prices Load forecasted Gas prices forecasted	1 year	**ANN**	- MAPE - MME - MeME	Spain (OMIE)
S12	(Alonso, Bastos, & García-Martos, 2016)	6.5 years (1 July 2006 – 31 December 2012)	Hourly (1h time interval)	Spot prices	2 months	**ARIMA**	- MAE - MedAE	Spain (OMIE)
S13	(Maciejowska & Weron, 2016)	4.5 years (22 April 2009 – 31 December 2013)	Half-hourly (30 minutes time interval)	Spot prices Natural gas price Electricity load Coal prices CO2 prices GDP	45 business days	**AR**	- MAPE - RMSE	UK (APX)
S14	(Yan, Song, & Chowdhury, 2016)	1 month (June 2009)	Hourly (1h time interval)	Spot Prices Natural Gas Price Hour Index Month Index Monthly Price Previous Year	1 month	**SVM** LSSVM	- MAE - MAPE	Pennsylvania, Jersey, Maryland (PJM)

Source: Own Work.

Concerning the documents type, they all are classified as articles, with the exception of one (Ziel & Steinert, 2018), that is also a review paper. As summarized in Figure 14, seven of the fourteen papers were published in 2016, five in 2019, and only two papers were published in 2017 and 2018. Regarding 2015, no relevant papers were found.

Figure 14 Number of mid-term ESPF papers per year



Source: Own Work.

In Figure 15, the research areas of the 14 papers are summarized. We can observe that the research areas related to mid-term ESPF are broad. The majority fits in energy fuels and various engineering fields.

Figure 15 Number of mid-term ESPF papers by research area

Source: Own Work.

When it comes to the window size of the training data used in the 14 selected studies, different sizes were chosen, going from just one month up to 12.5 years. Regarding the granularity of the data used as input for the models, ten papers worked with hourly resolution, while the other four with monthly resolution. There is no evidence of daily resolution approaches.

It is important to note that the studies S6, S7, S9 and S10 are probabilistic forecasts. Therefore, they have specific accuracy measures (Empirical Coverage Probability (ECP), Brier Score (BS), Winkler Score (WS) and Pinball Score/Loss (PBS)) and distinct techniques. Regarding the papers that present deterministic forecasts (papers: S1, S2, S3, S4, S5, S8, S11, S12, S13, and S14), the most used measures to access and compare the model's performance are: Mean Absolute Percentage Error (MAPE) in 7 of them; Mean Absolute Error (MAE) in 6 of them; Root Mean Square Error (RMSE) in 3 of them; and Mean Square Error (MSE) in 1 of them.

The deterministic forecast papers that used hourly resolution (papers: S1, S3, S5, S12, S13, and S14) perform forecasts from up to 1 month. With the exception of S12 (Alonso, Bastos, & García-Martos, 2016), that goes up to 2 months. The papers that used monthly data perform forecasts up to 1 month (papers: S4 and S8) or up to 1 year (papers: S2 and S11).

The models used to perform forecasting, for our mid-term ESPF case in the last 5 years (Table 1), are based either in Computational Intelligence (CI) or in Statistics.

Regarding the CI modeling approaches analyzed in the literature (papers: S3, S4, S5, S11 and S14), different techniques were adopted and compared. Some models being outperformed by others. The outperformed ones are described below.

Firstly, in 2016, a model based on Artificial Neural Networks was used by Ortiz, Ukar, Azevedo, & Múgica (2016). This paper lacks comparison with other models. In contrast, outperforming 9 different existing methods, Razak et al., (2019), develops and introduces a Least Square Support Vector Machine (LSSVM) with Bacterial Foraging Optimization

Algorithm (BFOA). The LSSVM and Support Vector Machine (SVM) approaches are also implemented in 2016 by (Yan, Song, & Chowdhury, 2016), where, between the two, SVM shows a better performance. In the same year, a Deep Long Short Term Memory network (DLSTM) based model, was adopted by Mujeeb et al. (2019). Windler, Busse, & Rieck, (2019) shows that Weights Nearest Neighbor (WNN), and Exponential Smoothing State Space Model with Box-Cox transformation, Autoregressive Integrated Moving Average errors, Trend and Seasonal components (TBATS) lead to very acceptable accuracies. Although, Deep Feedforward Neural Network (DFNN) achieved a slightly better accuracy.

When it comes to the **statistical** approaches to deal with mid-term ESPF, the models presented by the papers (S1, S2, S6, S7, S8, S9, S10, S12, S13) with better results are described below.

Alonso, Bastos, & García-Martos (2016) apply an Autoregressive Integrated Moving Average (ARIMA) model, adding forecast combination techniques. Maciejowska & Weron (2016) focus on Autoregressive models with and without fundamental variables, lacking a comparison with other models. Bello, Reneses, & Muñoz (2016) take advantage of the use of Logistic Regression and the Quadratic Equilibrium Model (QEM). Still in 2016, following what they have done before, Bello, Reneses, Muñoz, & Delgadillo (2016) use the QEM with the Generalized Additive Model for Location, Scale and Shape (GAMLSS). Also, continuing with what they had previously accomplished, (Bello, Bunn, Reneses, & Munoz, 2017) use Quantile Regression with the QEM. Differing from all the regression-based models used begore, Cheng, Luo, Miao, & Wu (2016) introduce a novel Grey Prediction Model with GM(0, N) interval, over the traditional GM(0, N) model. In 2018, Ziel & Steinert extend the X-Model, with the LASSO Regression and Bootstrap approaches. Ferreira, Ramos, & Fernandes (2019) perform a Multiple Linear Regression Model (MLRM), without comparing it with other models. More recently, Steinert & Ziel (2019) came up with a model that not only uses the precision of statistical autoregressive models, but go further by combining it with the market participant's prospects reflected in the future prices. They present an AR24-X model, where the X represents the external regressors.

Looking now a little further back, let us enumerate the models used in the papers from 2012 to 2014. Based on the literature review made by Ziel & Steinert (2018), the models implemented on the papers from that period were: SVM, WNN, Radial Basis Function (RBF), Seasonal ARIMA, Gaussian Mixture Models (GMM), K-Nearest Neighbor (k-NN), Multilayer Perceptron (MLP) and Linear Regression.

1.3.1 Models Overview

As seen in the literature review, different models and techniques were employed to forecast the electricity prices in a mid-term horizon, showing different levels of success. It is important to keep in mind that all those papers used different approaches and have different characteristics, making them unique and distinct studies. The performance of the used models cannot be easily replicated, since each case depends on the training data duration, market, country, economic cycle, data dimensions, time evolution, and other micro and macrostructure alterations, which turn comparisons hard. Also, even if all these different factors were controlled to be the same, the model's parameter settings can be set differently, giving distinct results.

It is very demanding to implement and compare all the analyzed models for price forecasting, due to memory capacity, CPU capacity, and time consumption. In this study, some of the most promising were implemented, to be tested and compared. There is only one paper using Portuguese data (paper: S2), in which only the MLRM model was implemented. So, all the models selected for this study were implemented for the first time using Portuguese data, as far as we know. For the purpose of achieving a broader and richer comparison, the models selected for our study were not only Computational Intelligence models, but also Statistical models, and some hybrid approaches too.

In this section, a brief presentation and description of each selected model is presented, so the reader can have a better understanding of the theory that lies behind each model.

Implemented Computational Intelligence Models:

- Autoregressive Neural Network (NNAR)
- Support Vector Machine (SVM)
- K-Nearest Neighbor (k-NN)
- Random Forest (RF)

Implemented Statistical Models:

- Autoregressive Integrated Moving Average (ARIMA)
- Exponential Smoothing (ETS)
- Linear Regression (LR) - LASSO, Elastic Net, and Ridge Regression
- Prophet

Implemented Hybrid Models:

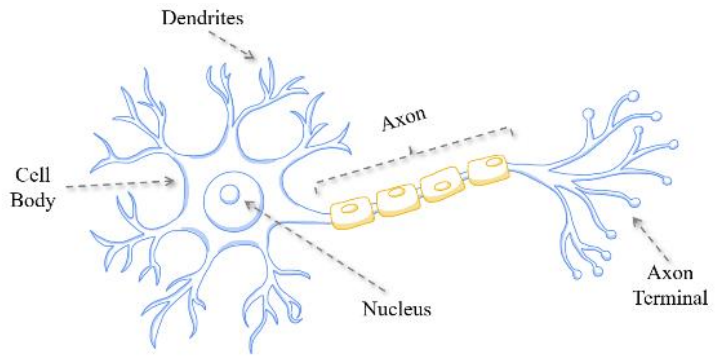
- ARIMA + Extreme Gradient Boosting (XGBoost)
- Prophet + Extreme Gradient Boosting (XGBoost)

1.3.1.1 Autoregressive Neural Network (NNAR)

Artificial Neural Networks (ANN) are computational data-driven models, characterized for their robustness, high tolerance to noisy data, flexibility, adaptability do deal with different data situations, generalization capabilities and collective computation (Yegnanarayana, 2006). The first ANN was created in 1958 by psychologist Frank Rosenblatt. ANN form the base of deep learning, a subsection of machine learning, where algorithms envisage to replicate and recreate the human brain and nervous system. They are called "neural" because they are based in neuroscience, although, traditional mathematical and statistical models are crucial for the ANN foundation (Hassoun, 1995).

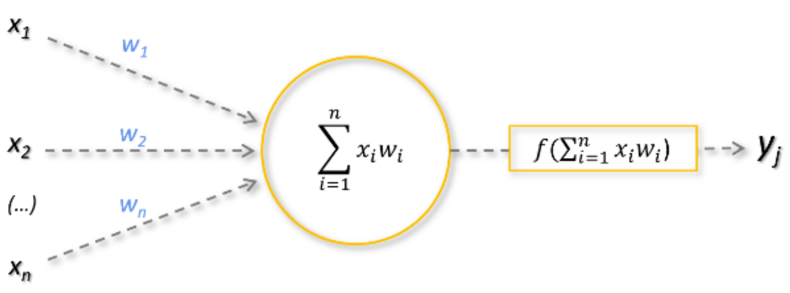
The human nervous system is formed by interconnected neurons. The biological neurons components are represented and identified in Figure 16. The terminal axon corresponds to the pre-synaptic region (output region). The synapse is where the neural activity is transmitted from neuron to neuron. See Figure 18. The neurons interconnection and weighted importance is extremely relevant for how the ANN are constituted (Graupe, 2013). See Figure 17. The analogy between biological neurons and artificial neurons lies in their form and how they connect between each other, where the dendrites and axon represents the nodes and the synapses represents the weighted connection (Mohammadhassani, Nezamabadi-Pour, Jumaat, Jameel, & Arumugam, 2013). ANN are formed and organized by layers of artificial neurons: the input layer, hidden layers and output layer. See Figure 19. They receive data in the input layer and, from that data, train themselves, creating patterns and predicting outcomes. The neurons (nodes/units) are the ones responsible for processing information. Most of the data is processed in the hidden layers. The connection between the units can be executed among units of the same layer (intralayer connection) or between units form different layers (interlayer connection) (Yegnanarayana, 2006). The data is propagated through the network. The values of each neurons correspond to a weight. On the output layers, the neuron with the highest value is the chosen one and is the output predicted value.

Figure 16 Biological neuron



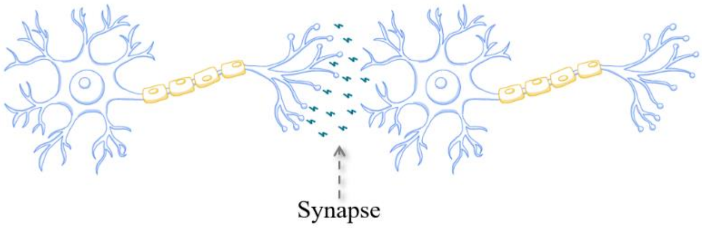
Source: Own Work.

Figure 17 Artificial neuron



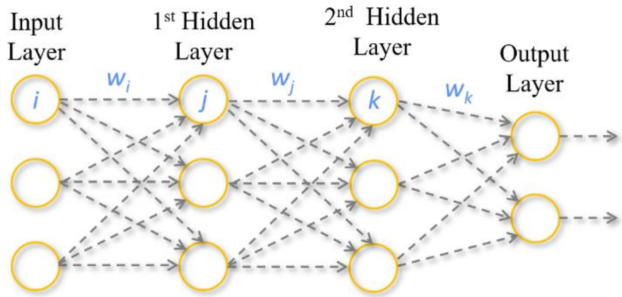
Source: Own Work.

Figure 18 Biological synapse



Source: Own Work.

Figure 19 ANN synapses



Source: Own Work.

The information in the ANN is processed through the multiplication of the inputs (x_1, x_2,…x_n) and the respective weights (w_1, w_2,…w_n). When all the inputs are multiplied by their weights, the sum is computed. The function *f* represents the activation function and is responsible for giving the output values of each neuron. This process is shown in Figure 17. That output can, in addition, serve as an input of another neuron in another layer.

The NNAR model is one type of ANN that is inspired by autoregression characteristics, using lagged values of time series as input to classify or predict sequences (Ramalheira, 2019). Generally is represented by *NNAR(p,k),* where p=lagged inputs and k=the number of nodes in the hidden model. But also by *NNAR(p, P, k),* when we are presented by a seasonal NNAR. NNAR is a feedforward neural network, which is formed by a linear combination function (equation (4)) and an activation function (equation (5)) (Thoplan, 2014).

$$net_j = \sum_i w_{ij} y_{ij} \tag{4}$$

$$f(y) = \frac{1}{1 + e^{-y}} \tag{5}$$

The *NNAR (p,0)* model corresponds to an ARIMA(p,0,0) model, without imposing any parameters restrictions to ensure stationarity (Maleki, Nasseri, Aminabad, & Hadi, 2018). However relying only on ANN for time-series forecast might not be the best solution, since they do not incorporate both linear and nonlinear behavior found in the real-world (Zhou et al., 2014).

1.3.1.2 Support Vector Machine (SVM)

Support Vector Machine (SVM) was introduced for the first time by Vladimir Vapnik (1995). SVM is a supervised model used for solving regression and classification problems, it can reduce data over-fitting and deals with high dimensional input spaces. In general, when compared to traditional models, SVM has obtained more robust and broad learning (Zhang, Wang, & Gao, 2019).

The three main differences, presented by Cao & Tay (2000), between SVM and other models rely on the fact that it uses linear functions set in a high dimension space, also estimates regression with risk minimization and, finally, uses a risk function and a regularization term based on the risk minimization principle (Cao & Tay, 2000). So, while SVMs applies the structural risk minimization (SRM) principle, ANN implements the empirical risk minimization (ERM) principle (Mohamed & El-Hawary, 2016). The generic SVR estimating function takes the form of the equation (6).

$$f(x) = \big(w * \Phi(x)\big) + b \qquad w \in X, b \in \mathcal{R} \tag{6}$$

Φ designates the transformation to a high dimensional space.

1.3.1.3 k-Nearest Neighbor (k-NN)

k-Nearest Neighbor (k-NN) was first introduced by Fix and Hodges Jr (1951). The basic idea of a k-NN application relies on the creation of groups for similar input objects (neighbors) in the training dataset, based on their features (Al-Qahtani & Crone, 2013). So, k-NN groups all the samples that evidence same properties categorized in the same feature space. Each group has the most similar k neighboring samples (Fan, Guo, Zheng, & Hong, 2019). For achieving that, k-NN calculates the distance between the points and then forms a set of groups based on the objects that evidence the closest distance between them. Metrics like the Euclidean distance or other types of distances are used in order to make that decision (Alkhatib, Najadat, Hmeidi, & Shatnawi, 2013). k-NN can be used for classification problems but also for regression and time-series forecasting. The idea of using k-NN as a time-series forecast model emerged in 1987 (Yakowitz, 1987). Time-series data often generates similar patterns, k-NN identifies those patterns and replicates their behavior into the future data. So, k-NN groups the k different similar patterns in the past data and, from them, a combination of future values emerges (Ban, Zhang, Pang, Sarrafzadeh, & Inoue, 2013).

1.3.1.4 *Random Forest (RF)*

Random Forest (RF) is a tree-based method introduced by Leo Breiman (2001). It is becoming widely popular and used for different applications including predictive modeling. This popularity is due to the fact that RF is very flexible to cope with missing values, handles categorical and continuous variables, generalization errors are calculated automatically, and the selection of the model's hyperparameters does not have a huge impact on the model performance (Aldrich, 2020).

RF consists on an ensemble of decision trees, by fitting multiple trees into a dataset and combining their results. Decision trees grow by choosing the best split dimension, considering all candidates/split options at each node, creating new branches. In regression, the RF prediction is the result of the average prediction values of each tree (Pórtoles, González, & Moguerza, 2018). One big advantage of using RF over just a decision trees model is the fact that decision trees tend to overfit (Hastie, Tibshirani, & Friedman, 2008).

1.3.1.5 *Autoregressive Integrated Moving Average (ARIMA)*

Introduced by Box and Jenkins in 1976, ARIMA became one of the most chosen models to forecast time-series (Masum, Liu, & Chiverton, 2018). The main reasons for that popularity came with the model's statistical characteristics, the capacity to apply several different exponential smoothing models, and the implementation of the Box-Jenkins methodology throughout the training procedure (Papastefanopoulos, Linardatos, & Kotsiantis, 2020). As the name implies, ARIMA is a model founded on both Moving Average and Autoregressive models. ARIMA uses a linear function to predict future values, based on past values and previous errors, on the basis of the assumption that a relationship between past observations and future values exists (Ramalheira, 2019).

Commonly nonseasonal ARIMA is represented by *ARIMA(p,d,q)*, where p= autoregressive order, d=degree of differentiation, and q= the moving average order. Seasonal ARIMA is denoted by *ARIMA(p, d, q) (P, D, Q)m*, where m= number of periods in each season, P= autoregressive, D=differencing, and Q=moving average, for the seasonal part of the ARIMA model. The model is defined by the equation (7).

$$\left(1-\sum_{i=1}^{p}\left(\Phi_i L^i\right)\right)(1-L)^d X_t = (1-\sum_{j=1}^{q}\left(\Phi_i L^j\right)\mathcal{E}_t) \tag{7}$$

Where L is the offset.

ARIMA is based on the premise that the data used is statistically stationary (i.e. mean, variance, and autocorrelation are all constant over time). If not, the model applies a differentiation operator to transform the time series (Al-Musaylh, Deo, Adamowski, & Li, 2018).

1.3.1.6 Exponential Smoothing (ETS)

Exponential smoothing (ETS) was first suggested by Robert Goodell Brown in 1956. It is a a model that weighs observed values of historical data, in order to forecast future values, taking into consideration time variation (Zhang, Wang, & Gao, 2019). The ETS model considers that time series are formed by three main components: the Error (E), the trend (T), and the Seasonal components, which can be additive (A), multiplicative (M) or none (N) (Yang, Sharma, Ye, Lim, Zhao, & Aryaputera, 2015). According to Panigrahi & Behera (2017) the general model entails a state vector $x_t = (l_t, b_t, s_t, s_{t-1}, \ldots, s_{t-m+1})'$ and the state space equations are represented by equations (8) and (9).

$$y_t = w(x_{t-1}) + r(x_{t-1})\mathcal{E}_t \tag{8}$$

$$y_t = w(x_{t-1}) + r(x_{t-1})\mathcal{E}_t \tag{9}$$

1.3.1.7 Linear Regression (LASSO, Elastic Net, and Ridge Regression)

Linear regression is one of the simplest and broadest used methods for predictive models. The concept of regression was first introduced by Francis Galton (1885). It can be formulated as the following equation (10).

$$\hat{\boldsymbol{y}} = \hat{\beta}_0 + x_1\hat{\beta}_1 + \cdots + \ x_p\hat{\beta}_p \tag{10}$$

Where Y is the dependent variable to be predicted, $(x_1, \ldots, x_p)$ represents the independent variables, and the vector $\widehat{\boldsymbol{\beta}} = (\hat{\beta}_0, \ldots, \hat{\beta}_p)$ are the coefficients, that attribute weights to the features, based on their importance.

LASSO Regression

LASSO (Least Absolute Shrinkage and Selection Operator) regression was first introduced by Tibshirani in 1996. It comes with two main beneficial functions: regularization and variable selection (Tang, Mao, Wang, & Nelms, 2018). This model sets the values of the regression coefficients ($\widehat{\boldsymbol{\beta}}$) to zero when the respective variables have low importance. Once a variable has a 0 coefficient, it has no impact on the model anymore, resulting in its removal. Therefore, the model uses only a few variables, having a sparse solution (Tang, Mao, Wang, & Nelms, 2018). It uses a L1 penalty.

Ridge Regression

Ridge regression was developed by Horel and Kennard in 1970. It is similar to a LASSO model, but unlike LASSO, Ridge Regression tends to compress less relevant parameters for values close to zero, although never zero (Hao, Zhao, & Wang, 2020). It uses a L2 penalty.

Several experiments have been made in order to compare LASSO and Ridge regression, concluding that neither of them can clearly dominate the other (J. Li & Chen, 2014).

Elastic Net

LASSO and Ridge Regression can be considered as special cases of Elastic Net, developed by Zou & Hastie (2005). Elastic Net avoids the extreme approaches used by LASSO and Ridge Regression, combining both shrinkage and selection approaches instead (Hao, Zhao, & Wang, 2020).

The penalty of elastic net consists on the mixture between penalty L1 and L2 (Pereira, Basto, & Silva, 2016) and it can be formulated by equation (11).

$$P_{\alpha} = \sum_{i=1}^{p} [\frac{1}{2}(1-\alpha)\beta_j^2 + \alpha|\beta_j|] \tag{11}$$

Elastic net can be simplified as a LASSO model when α is set as zero and to a ridge regression when α is equal to one (Ogutu, Schulz-Streeck, & Piepho, 2012).

1.3.1.8 Prophet

Prophet is a new and promising model created by Facebook, introduced by Taylor & Letham (2018). It consists of a combination of an additive model and fitting trends, and seasonal components together. Prophet has proven good results in the literature when forecasting time-series. It is evidenced as not being sensitive to missing data, having robust characteristics, shifts in the trend and large outliers (Aguilera, Guardiola-Albert, Naranjo-Fernández, & Kohfahl, 2019)

Prophet is based on an a Generalized additive model (GAMs) (Hastie & Tibshirani, 1987). GAMs allow to model complex patterns, thanks to the sums of smooth functions. Prophet is established with three main components represented by equation (12).

$$y(t) = g(t) + s(t) + h(t) + \varepsilon_t \tag{12}$$

Where *g(t)* represents the trend function that models non periodic linear and logistic regression changes in the time-series, *s(t)* corresponds to the periodic components (i.e. weeks, months, years…), *h(t)* is the effect of holidays in the values, and ε_t is the error.

1.3.1.9 Gradient Boosting (XGBoost)

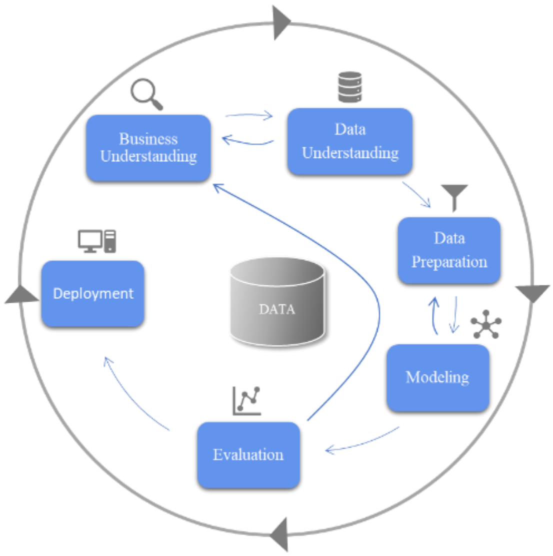
Extreme gradient boosting (XGboost), proposed by Friedman (2000), is a scalable system used for tree boosting, designed for speed and performance (P. Li & Zhang, 2018). The goal of the algorithm is to achieve the optimized values of the objective function and minimize the error (Zheng, Yuan, & Chen, 2017). It is used for improving models' performance.

In this study, XGBoost is used to forecast the residuals of the ARIMA and Prophet model, then, the forecasting output for each time series is accomplished from the sum of the ARIMA/Prophet and XGBoost models.

2 DATA AND METHODOLOGY

The methodology used to implement the intelligent computing models to forecast electricity spot prices in this thesis is based on the CRISP-DM (Figure 20), Cross-Industry Standard Process for Data Mining (Wirth, 2000). It offers a powerful guidance for even the most advanced data science activities that are gaining adoption in our society and our economy. It consists in six phases: business understanding, data understanding, data preparation, modeling, evaluation, and deployment. We can map the first phase, business understanding, with the *Introduction* and the *Research Background and Literature Review* chapters of the thesis. The remaining phases are mapped in this chapter *Data and Methodology* and the next, *Results and Discussion*. The deployment phase was left out of this study.

Figure 20 The Data Mining Process, according to the CRISP-DM methodology



Source: Own Work.

Our data analysis and machine learning modeling were performed using R software version 3.6.3, and Python and Pandas software version 6.0.1. Packages and extensions used in R, were: 'modeltime', 'timetk', 'tidymodels', 'lubridate', 'tidyverse', 'randomForest', 'glmnet', 'kernlab', 'kknn', 'nnet', and 'modeltime.ensemble'. Packages and libraries used in Python were the following: 'pandas', 'numpy', 'seaborn', 'sklearn', 'scipy', 'statsmodels.api', and 'matplotlib.pyplot'.

2.1 Data Understanding & Preparation

Before the Machine Learning models were ready to be created, trained, tested, and compared, the data was explored, analyzed, and pre-processed.

2.1.1 Data Description and Quality

The hourly data set regarding the Portuguese electricity spot prices is publicly available and was extracted from the OMIE website (https://www.omie.es/pt/file-access-list). The OMIE files (*.csv* format) were obtained for each day of the year in the period between July 2007 and December 2019. Each file represents a day of the year and contains the price values for each hour of that same day. All those files were imported, read, integrated and merged resulting in one dataset with 2 features: date as index (format: "*yyyy-mm-dd HH:MM:SS*") and the price value (€/MWh).

In order to be able to make a proper analysis and forecast, we started by assessing the quality of the data. For that, we checked for missing values, wrong data, duplicate data, the range of the data and outliers. There were some missing values and wrong values regarding the prices of the years between 2007 till 2013. After contacting OMIE, we managed to fix those issues with the correct data. Since there is no data for the first half of 2007, all the prices regarding this year were not considered and deleted. After that, the duplicates and empty rows were removed. In Spain and Portugal, unlike most of European power markets, day-ahead offers must range between 0 and 180.3 €/MWh and therefore, negative prices are not allowed. In the dataset, we did not find any negative values for the hourly prices and the max value was identified as 180.3€/MWh. So, the prices in the dataset lie in the expected interval: [0, 180.3].

At this point, dataset pre-processing was complete and correct, with 105192 rows × 2 columns. Each row representing an hour from the period between 2008 to 2019 (12 years). Since this study envisages to forecast the ESP using two different approaches (daily and monthly), two new datasets were created. In the first one, the hourly prices were transformed into daily prices, by averaging the values by the day of the year, resulting in a dataset with 4383 rows × 2 columns. The date format was set to: "*yyyy-mm-dd*". In the second data set, the values were averaged by month of the year, resulting on a dataset with 144 rows × 2 columns. The date format was set to: "*yyyy-mm*".

The description of the prices is summarized in Table 2 and the frequency distribution of the daily prices is represented in

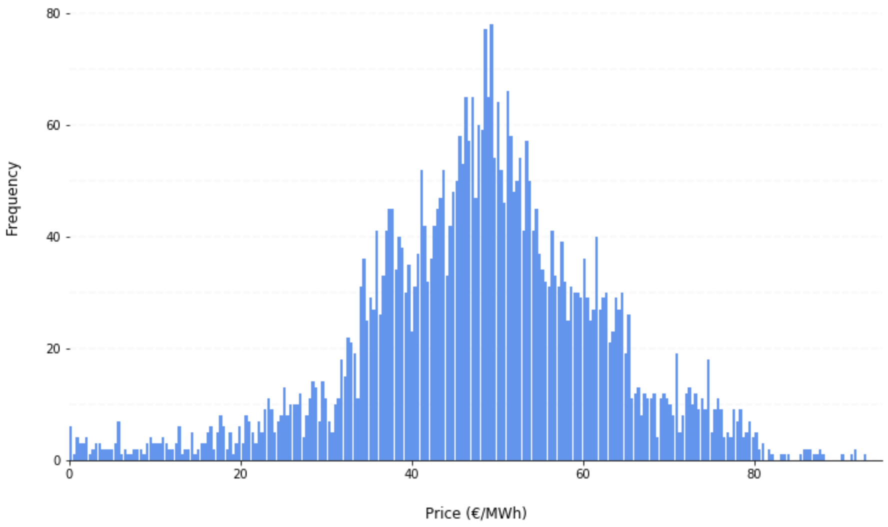
Figure 21. The mean during those 12 years is 48.05€/MWh and the standard deviation is 14€/MWh. December 8th, 2013 was the day that averaged the highest value (93.11€/MWh). March 29th, 2013 was the day that averaged the lowest value for the price (0 €/MWh).

Table 2 Basic descriptive statistics of the portuguese ESP (€/MWh)

Mean	Std.	Min.	25% (1st Qu.)	50% (Median)	75% (3rd Qu.)	Max.
48.05	14.13	0	39.96	48.53	56.61	93.11

Source: Own Work.

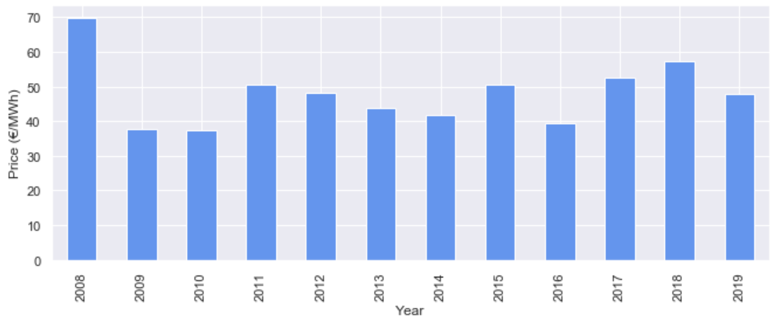
Figure 21 Frequency distribution of the portuguese ESP



Source: Own Work.

The price average of each year is plotted in Figure 22. The year with the highest price average is 2008 (70.90€/MWh). In contrast, 2009 was the year with the lowest average (36.94€/MWh). The average of the prices does not change that much from year to year.

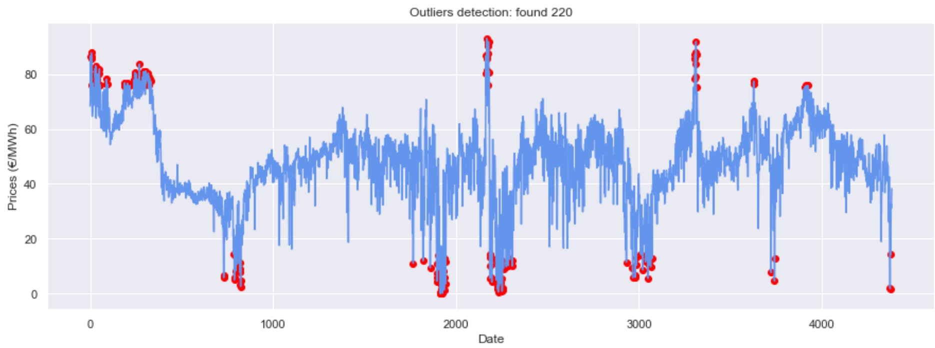
Figure 22 Average of the portuguese ESP by year



Source: Own Work.

The time-series of the daily prices is represented in Figure 23, with the found outliers in red dots. This series presents high volatility, temporary spikes and frequent extreme values. Even though 220 outliers were identified, no operation was performed to remove or change them, because, in this study, abnormal values reflect the actual nature of the prices and by manipulating them, could lead to lose its informative feature. The outliers with higher value can be called as *price spikes*. The *spikes* correspond to periods of unexpected high electricity demand, resulting in extreme fluctuations on spot prices.

Figure 23 Outliers of the portugue ESP time-series

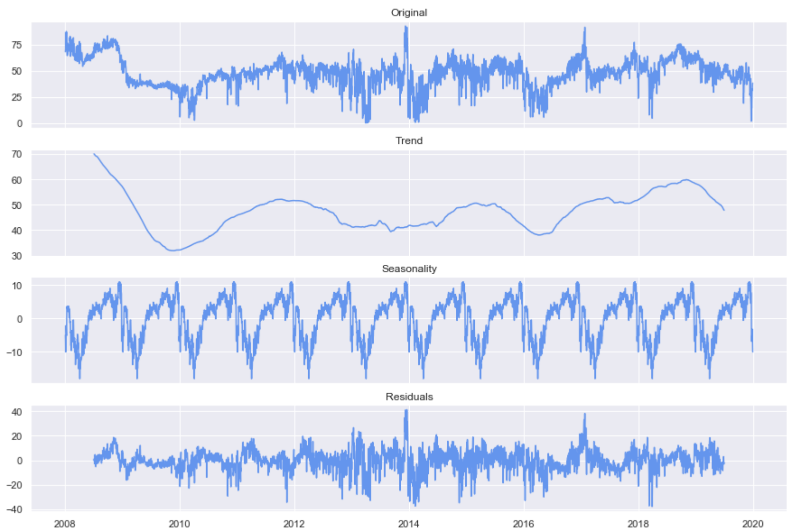


Source: Own Work.

2.1.2 Time-Series Analysis

A useful approach to get insights into the data is to decompose the original time-series (Figure 24 - first panel). STL - Seasonal and Trend decomposition using Loess (Cleveland, McRae, & Terpenning, 1990) is a versatile and robust method for decomposing time-series. This decomposition results in the *trend*, *seasonality* and *residuals* components (Figure 24 - second, third and fourth panel respectively). The *trend* captures the slowly moving overall level of the series, thanks to the application of a one year rolling mean. The *seasonality* captures patterns that repeat every season. The *residuals* represent what is left (*residuals* = *original* - *trend* - *seasonality*.).

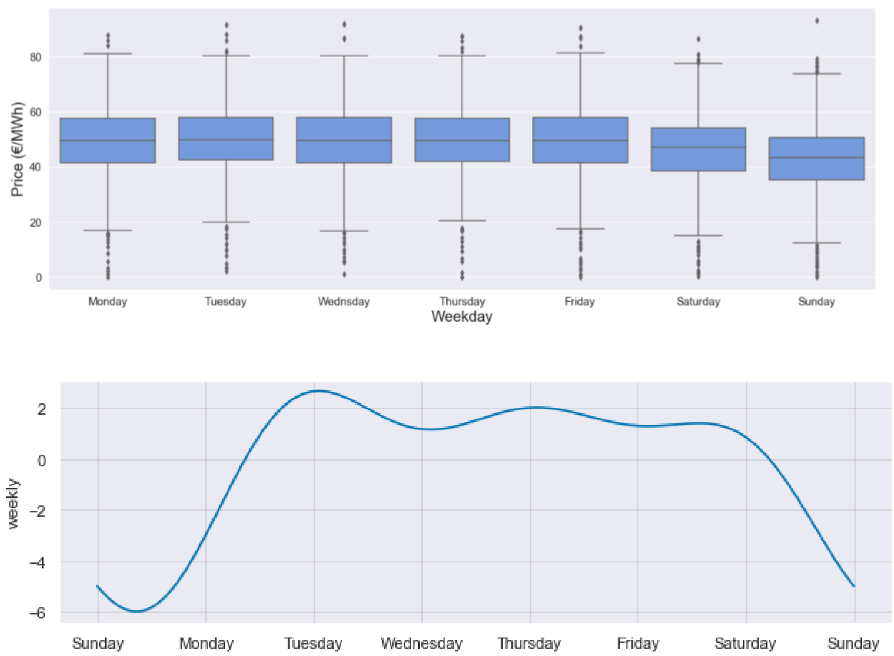
Figure 24 Portuguese ESP time-serie decompositions



Source: Own Work.

The *seasonality* can be better understood with a *seasonal diagnostic* represented in Figure 25 and Figure 26. This diagnostic provides a deeper understanding of the price's behavior during the year and the week. It compares fluctuations of the data of different days and months. From Figure 25, the prices show an upward trend on Tuesday and a downward trend on Sunday. The weekdays show very similar patterns, the median of the prices of each weekday is constant (around 49€/MWh) and the upper and lower fence of each box plot are alike. In contrast, in the weekends, we have lower values for the fences and median. One possible reason for this event could be the fact that big companies, businesses, and some stores are closed during this time. Sunday presents the lowest prices on average (43.34€/MWh).

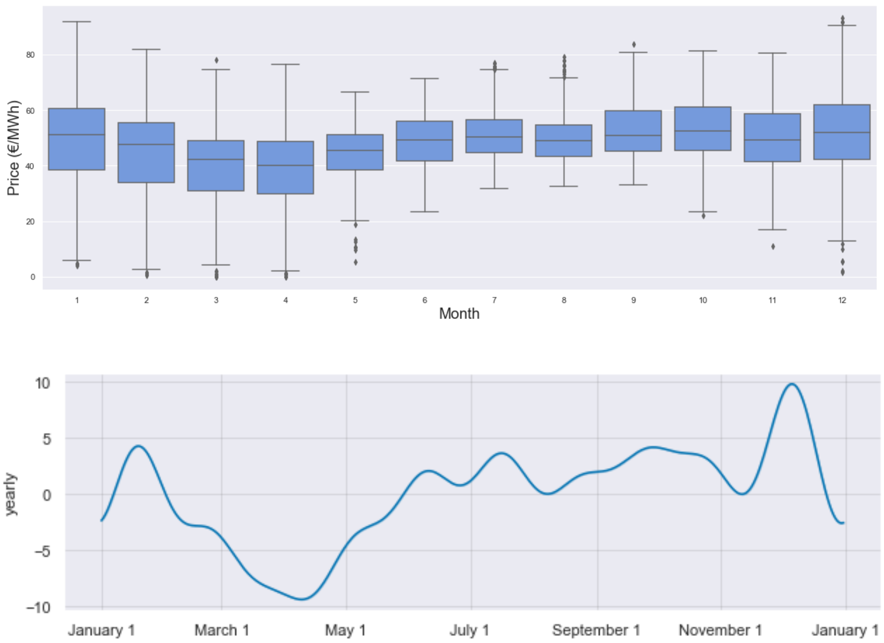
Figure 25 Seasonal diagnostic – Week



Source: Own Work.

In Figure 26, the prices showed a downward trend in February, March and April, while December shows the highest peak of the year. The boxplot of December has the highest upper fence and median (52.06€/MWh). While February is the month with the lowest lower fence and April the month with lowest median (40.12€/MWh). Prices during colder months (from October to April), tend to have a much higher standard deviation/volatility when compared to prices during hotter months (from May to September).

Figure 26 Seasonal diagnostic - Year



Source: Own Work.

Another relevant component of a time series is the *stationarity*. This is fundamental to be analyzed, since some forecast models assume the stationarity condition. A time-series is considered stationary when it progresses randomly around a constant average, showing some stable equilibrium.

The Augmented Dickey-Fuller test (ADF) (Dickey & Fuller, 1979) is a type of statistical test used to verify if a given time series is stationary. This test was performed in Python. Below, in

Table 3, the results are shown.

AUGMENTED DICKEY-FULLER TEST	
Null Hypothesis (H0):	The time-series is not stationary
Alternate Hypothesis (H1):	The time-series is stationary
ADF Statistic:	-4.292631
P-Value:	0.000457
Critical Values:	**1%:** -3.432 **5%:** -2.862 **10%:** -2.567

Table 3 Augmented dickey-fuller test (ADF)

Source: Own Work.

We can see that our statistic value of -4.29 is less than the value of -3.432 at 1% and the p-value is below the threshold (0.10, 0.05, and 0.01). This suggests that we can reject the null hypothesis with a significance level of less than 1%. Therefore, the time-series seems to be stationary.

2.1.3 External Factors

In this study, it was important to understand if external factors impact the electricity spot prices, not only to better understand the ESP itself, but also to see if those factors could serve as input for the models. Variables like the level of load, level of generation and weather are the most used in the literature as input for ESPF models.

Weather variables might not be a reliable input for a one year ahead ESPF, since a reliable, useful and accurate forecast of the weather can only go up to 3 to 10 days into the future (Bauer, Thorpe, & Brunet, 2015). Daily temperature forecasts for the next 365 days have enormous potential error in perspective. In the literature, models that use forecast weather data as an input have a scope where the forecast horizon only goes up to a maximum of one month.

The impact of *electric load and generation variables* was tested by calculating their correlation with the prices. The historical data (from 2007 till 2019) of the Portuguese load and generation was retrieved from the REN website (www.mercado.ren.pt) and then a Pearson correlation between them and the prices was computed and evaluated by calculating the correlation coefficient (Equation (13)).

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2(y_i - \bar{y})^2}} \tag{13}$$

$$where,$$

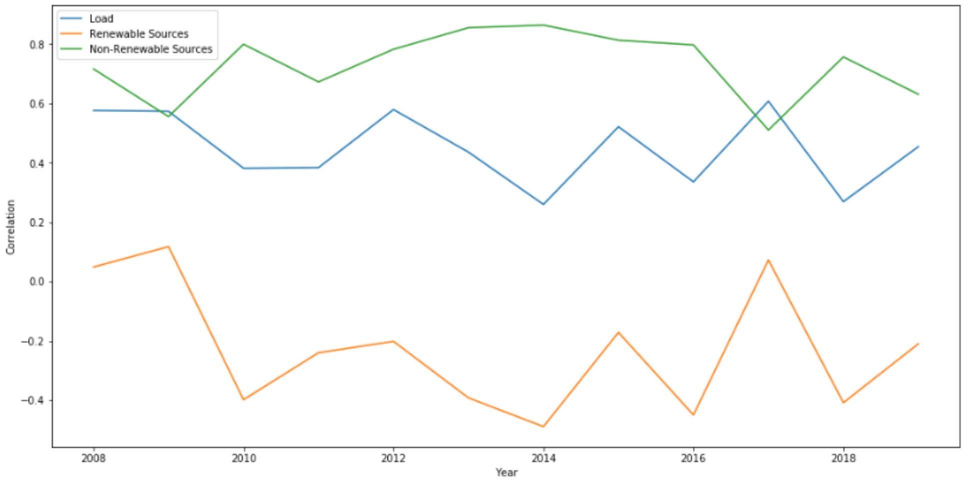
$$r = correlation\ coeficient$$

$$x_i = values\ of\ the\ x\ variable\ in\ a\ sample$$

$$y_i = values\ of\ the\ y\ variable\ in\ a\ sample$$

Regarding the results of the Pearson correlation (during the past 12 years), we found that the Pearson coefficient between the load and the price is 0.354, which means that there is a positive weak correlation between them. Between the renewable sources and prices, we found a correlation of -0.334, corresponding to a negative weak correlation between them. Finally, the Pearson coefficient between the non-renewable sources and prices is 0.624, showing that there is a positive moderate/sufficient correlation between them. Figure 27 shows the result of the correlations evaluated, by year.

Figure 27 Correlation between the Prices with the Load and the Generation variables



Source: Own Work.

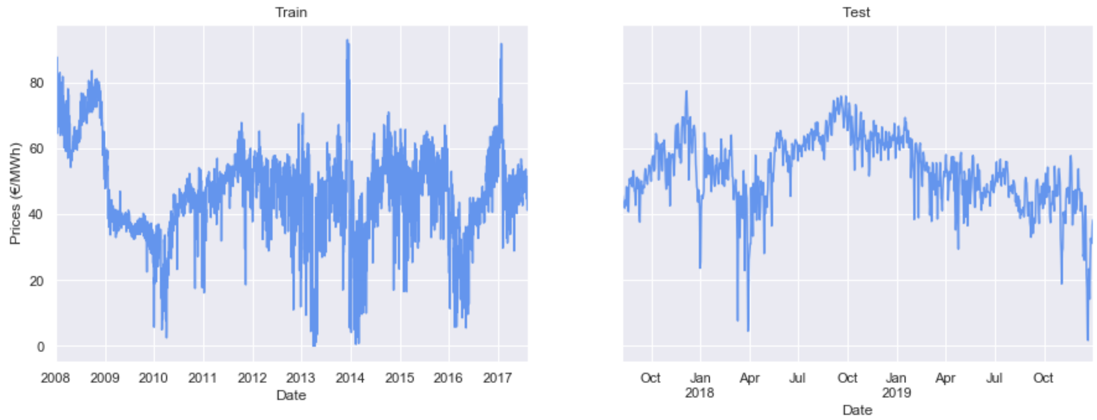
So, we can conclude the load and generation variables can somehow impact the ESP, although they were not used as input features for the models in this study, due to their forecast complexity. The mid-term forecast of the electric load and generation levels is a very complex and difficult task (Mir, Alghassab, Ullah, Khan, Lu, & Imran, 2020). There are different types of renewable and non-renewable electricity production sources: Hydro, Wind, Biomass, Solar, Coal, Natural Gas, etc. All those different sources must be studied separately in order to be forecasted and to achieve acceptable and reliable forecasts for the total generation levels. Forecasting all of those variables goes out of the scope of this study.

In addition, adding external forecasted variables in the model can be a risk, since it can lead to a larger error component (Weron, 2014). Therefore, in this study, it was assumed that general fluctuations in energy demand, energy generation and temperature are already incorporated in historical electricity prices.

2.1.4 Train/Test Set

Our time-series data was split into training and testing sets, according to the adopted CRISP-DM methodology (Figure 28). The training set is the sample of data utilized to fit the machine learning model. It corresponds to 80% of the data and goes from 2008-01-01 to 2017-08-06. The test set is the sample of data utilized to evaluate the model that was trained with the training dataset, delivering an impartial and fair evaluation, while tuning the models' hyperparameters. It corresponds to 20% of the data and is mutually exclusive of training set, ranging from 2017-08-07 to 2019-12-31.

Figure 28 Train set and Test set



Source: Own Work.

2.1.5 Feature Engineering

Before creating, fitting, and training the forecast models it is important to create features from the existing data as input for the models. This process, called feature engineering, consists in extract knowledge from existing data.

First, 15 features extracted from the *date* were created, based on the day, month, week and year. They are described in Table 4.

Then in order to handle periodic variables in the forecasting models, we applied sine and cosine transformations on the number of time steps elapsed since the beginning of each seasonal period. Therefore, such Fourier features were created to model seasonality. For that purpose, *period* and *order* had to be set.

The *period* corresponds to the numeric period of the oscillation frequency. Since we are dealing with daily data, three types of frequency were specified:

- Yearly frequency: 365
- Quarterly frequency: 365 / 4 = 91.25
- Monthly frequency: 365 / 12 = 30.42

K is the number of orders that each sine/cosine Fourier series has. So, the number of complete waves in the interval $[-\pi,\pi]$ is represented by *K*. In this study we decided to specify K as 5, because the increase of the *K* leads to the model's better ability to fit. Although, it can also come with the risk of overfitting. Therefore, 5 is a good choice when balancing performance with reliability. Fourier features were created by implementing: *step_fourier(period =*

c(30.42,91.25, 365), K = 5. This scheme returned 30 Fourier series that are described in Table 4.

For the monthly approach, the same was done but considering the monthly data resolution characteristics.

Table 4 Features created as input for the models

NAME	DESCRIPTION	POSSIBLE VALUES	TYPE
DATE FEATURES			
year	Represents the calendar year.	2008 to 2019.	Integer
year ISO	The ISO year number of the year (Monday start)	2008 to 2019.	Integer
half	Represents the half of the year.	1 or 2.	Integer
quarter	Represents the quarter of the year.	1 to 4	Integer
month	Represents each month of the year.	1 to 12	Integer
day	Represents each day of the month.	1 to 31	Integer
wday	Represents each day of the week.	1 to 7	Integer
qday	Represents each day of the quarter of a year.	1 to 91	Integer
yday	Represents each day of a year.	1 to 365	Integer
mweek	Represents the week of the month.	1 to 5	Integer
week	Represents the week of the year.	1 to 51	Integer
week2	Represents the modulus for bi-weekly frequency.	0 or 1	Integer
week3	Represents the modulus for tri-weekly frequency.	0 to 2	Integer
week4	Represents the modulus for quad-weekly frequency.	0 to 3	Integer
mday7	Identifies the instance/order in which that the day of the week has appeared in the month.	1 to 5	Integer
FOURIER FEATURES			
cos365_K1	Cosine with period = 365 and K = 1	-1 to 1	Double
sin365_K1	Sine with period = 365 and K = 1	-1 to 1	Double
cos365_K2	Cosine with period = 365 and K = 2	-1 to 1	Double
sin365_K2	Sine with period = 365 and K = 2	-1 to 1	Double
cos365_K3	Cosine with period = 365 and K = 3	-1 to 1	Double
sin365_K3	Sine with period = 365 and K = 3	-1 to 1	Double
cos365_K4	Cosine with period = 365 and K = 4	-1 to 1	Double
sin365_K4	Sine with period = 365 and K = 4	-1 to 1	Double
cos365_K5	Cosine with period = 365 and K = 5	-1 to 1	Double
sin365_K5	Sine with period = 365 and K = 5	-1 to 1	Double
cos91.25_K1	Cosine with period = 91.25 and K = 1	-1 to 1	Double
sin91.25_K1	Sine with period = 91.25 and K = 1	-1 to 1	Double
cos91.25_K2	Cosine with period = 91.25 and K = 2	-1 to 1	Double
sin91.25_K2	Sine with period = 91.25 and K = 2	-1 to 1	Double
cos91.25_K3	Cosine with period = 91.25 and K = 3	-1 to 1	Double
sin91.25_K3	Sine with period = 91.25 and K = 3	-1 to 1	Double
cos91.25_K4	Cosine with period = 91.25 and K = 4	-1 to 1	Double
sin91.25_K4	Sine with period = 91.25 and K = 4	-1 to 1	Double
cos91.25_K5	Cosine with period = 91.25 and K = 5	-1 to 1	Double
sin91.25_K5	Sine with period = 91.25 and K = 5	-1 to 1	Double
cos30.42_K1	Cosine with period = 30.42and K = 1	-1 to 1	Double
sin30.42_K1	Sine with period = 30.42and K = 1	-1 to 1	Double
cos30.42_K2	Cosine with period = 30.42and K = 2	-1 to 1	Double
sin30.42_K2	Sine with period = 30.42 and K = 2	-1 to 1	Double

Source: Own Work.

cos30.42_K3	Cosine with period = 30.42 and K = 3	-1 to 1	Double
sin30.42_K3	Sine with period = 30.42 and K = 3	-1 to 1	Double
cos30.42_K4	Cosine with period = 30.42 and K = 4	-1 to 1	Double
sin30.42_K4	Sine with period = 30.42 and K = 4	-1 to 1	Double
cos30.42_K5	Cosine with period = 30.42 and K = 5	-1 to 1	Double
sin30.42_K5	Sine with period = 30.42 and K = 5	-1 to 1	Double

2.2 Models Implementation

After the data was explored, analyzed, and pre-processed, the Machine Learning models were ready to be created, trained, tested, and compared.

2.2.1 Models Computation

The models were computed in the R programming environment, using mainly the *forecast* and *modeltime* packages. The theory behind each model can be read in section *2.1.6 Models Overview*.

ARIMA, Prophet, and ETS were computed via the following main steps:

1. Implementation of the specification function, where the general model algorithm and the respective parameters were set up.
2. Set of the engine, where the specific package-function to use was selected.
3. Fit of the model to the data, where the *date* column was set to be a regressor.

These three models are the only ones that do not use the features created in the feature engineering preprocess, i.e., they only use the *date* and the *price* columns as input.

The remaining models (NNAR, SVM, k-NN, RF, ARIMA Boost, Prophet Boost, and LR) are more complex, requiring a ***workflow***. These models were computed through the following steps:

1. Creation of the model specification, where the specification function was implemented, and the engine was set.
2. Manual selection and tuning of the parameter's values (with the exception of the ARIMA Boost and Prophet Boost, since their parameters were automatically set by the packages in R).
3. Creation of a workflow, where the model's specifications and the pre-processed features were added.
4. Fitting the workflow to the data.

Table 5 represents a summary of each created model, respective parameters to tune, the function, engine and mode of analysis.

Table 5 Summary of the models implemented to forecast the portuguese ESP

MODEL	PARAMETERS	FUNCTION	ENGINE	MODE
ARIMA	AUTO	arima_reg()	auto_arima	Regression
Prophet	AUTO	prophet_reg()	prophet	Regression
Exponential Smoothing	AUTO	exp_smoothing()	ets	Regression
Linear Regression: -LASSO -Elastic Net -Ridge Regression	- Penalty - Mixture	linear_reg()	glmnet	Regression
Random Forest	- Mtry (F) - Trees (K) - Min Node	rand_forest()	randomForest	Regression
Support Vector Machine	- Cost - Sigma	svm_rbf()	Kernlab	Regression
k-Nearest Neighbor	- Neighbors - Distance Type	nearest_neighbor()	kknn	Regression
Neural Network Autoregression	- Non seasonal AR (p) - Seasonal AR (P) - Hidden units (K) - Epochs	nnetar_reg()	nnetar	Regression
Prophet Boost	AUTO	arima_boost()	auto_arima_xgboost	Regression
ARIMA Boost	AUTO	prophet_boost()	prophet_xgboost	Regression

Source: Own Work.

2.2.2 Parameters Tuning

As seen in Table 5, the models have different parameters that can and must be tuned, for the purpose of achieving the most accurate predictions. The parameters of ARIMA, Prophet, Exponential Smoothing, Prophet Boost, and ARIMA Boost were automatically optimized and selected thanks to the packages and respective libraries used in R. The parameters of the remaining models had to be manually selected. The procedure of picking among different parameters is called model "tuning". Since there is not an available analytical model to find the optimal combination, the solution lies on experimentally testing different combinations. Once testing all the possible values and combinations of values for all the model's parameters is impractical and time-consuming, a simpler and reliable solution is using a *grid* *search* (Bergstra & Bengio, 2012). *Grid search* is the most widely implemented scheme for parameter optimization (Bergstra & Bengio, 2012). Therefore, to provide robust empirical results, such *grid search* approach was used to find the optimal values for the model's parameters. With such a scheme, not all the possible values are tested but, instead, jumps of values are established and tested. Combinations of parameter's values were performed, tested, and compared as individual models.

It is important to note that as a result of the random nature of some models like RF and NNAR, the random seed had to be defined. This was done in order to train different models with the same starting point (same seed) and compare the results when the parameters changed. The random seed was fixed to 15, making the initial starting weights of the models the same each time. Consequently, any variation in the performance of the model can be attributed to the parameter tunning.

Parameters of Linear Regression

- ***Mixture:*** A number between zero and one (inclusive) corresponding to the proportion of regularization in the model. When mixture = 1, it is a pure *LASSO* model while mixture = 0 indicates that *ridge regression* is being used. To implement an *elastic net* the mixture must be set between 0 and 1. In this study it was set as 0.5. Values tested (for monthly and daily data) = {0, 0.5, 1}
- ***Penalty***: A positive number representing the total amount of regularization. There is no equation for finding the best penalty value. Thus, we needed to iterate a series of values and evaluate prediction performances. In this study jumps of values were tested till the accuracy kept constant. Values tested (for monthly and daily data) = {0.01, 0.5, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 100, 200, 1000, 2000, 5000, 6000, 7000}.

Parameters to tune on Random Forest

- ***Mtry (F)***: An integer of input variables (predictors) that will be randomly sampled at each split when creating the tree models. The RF procedure is not overly sensitive to the value of F. The value for this parameter is recommended to be set as one-third of the predictors for regression. Value used (monthly data) = 15. Value used (daily data) = 21.
- ***Trees (K)***: An integer of the trees contained in the ensemble. Successive trees must be experimented until the error stabilizes. Values tested (for monthly and daily data) = {1, 2, 3, 4, 5, 6, 8, 10, 20, 30, 40, 50, 100, 200, 300, 400, 5000, 6000}.
- ***Min_n***: An integer for the minimum of data points in a node that are required for the node to be split. RF also shows low sensitivity to this parameter. Since the default value of the function is five, and since some of the most relevant authors also recommend to set it as 5: Value used (for monthly and data) = 5.

Parameters to tune on SVM

- ***cost:*** A positive number for the cost of predicting a sample within or on the wrong side of the margin. It is a hypermeter in SVM to control error. Values tested (for monthly and daily data) = {0.001, 0.01, 0.1, 1, 10, 100}.
- ***rbf_sigma:*** A positive number for radial basis function. Values tested (for monthly and daily data) = {0.001, 0.01, 0.1, 1, 10, 100}.

Parameters to tune on k-NN

- ***neighbors (k):*** A single integer for the number of neighbors to consider. I set K min = 2, to avoid overfitting and K max = 100 (since the model does not support a larger value than 100 for K). Values tested (for monthly and daily data) = {2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 25, 30, 40, 50, 60, 80, 100}.
- ***dist_power:*** The parameter used when calculating the Minkowski distance. It can be specifically set as the Manhattan distance (set value = 1) and the Euclidean distance (set value = 2). Values tested (for monthly and daily data) = {1, 2}

Parameters to tune on NNAR

- ***non_seasonal_ar (p):*** The order of the non-seasonal auto-regressive (AR) terms. Values tested (for monthly data) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}. Values tested (for daily data) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}.
- ***seasonal_ar (P):*** The order of the seasonal auto-regressive (SAR) terms. The P was set to zero when using monthly data, since the ARIMA and ETS models did not consider the seasonality. Value used (for monthly data) = {0}. Values tested (for daily data) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}.
- ***hidden_units (k):*** An integer for the number of units in the hidden model. Values tested (for monthly data) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}. Values tested (for daily data) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}.
- ***epochs:*** An integer for the number of training iterations. Value used (for monthly data) = 300. Value used (for daily data) = 100.

2.2.3 Error Measurements

To measure the quality of the predictions and assess the performance of the forecasting models, accuracy metrics were implemented. With these metrics, the expected value and predicted value are compared, to assess the error in the prediction. The magnitude of the error translates the accuracy of the model's predictions. In this study, the selected and implemented metrics are: Mean Absolute Error (MAE) (Equation (14)), Mean Absolute Percentage Error (MAPE) (Equation (15)), Mean Absolute Scaled Error (MASE) (Equation

(16)), Symmetric Mean Absolute Percentage Error (SMAPE) (Equation (17)), and Root Mean Squared Error (RMSE) (Equation (18)). The underlying expressions of these accuracy metrics are given by:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|y_i - \hat{y}_i| \tag{14}$$

$$MAPE = \frac{100}{N}\sum_{i=1}^{N}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \tag{15}$$

$$MASE = \frac{MAE}{\frac{1}{N-1}\sum_{i=2}^{N}|y_i - y_{i-1}|} \tag{16}$$

$$SMAPE = \frac{100}{N}\sum_{i=1}^{N}\frac{|\hat{y}_i - y_i|}{(|y_i| + |\hat{y}_i|)/2} \tag{17}$$

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2} \tag{18}$$

$$where,$$

$$y_i - observed\ value\ at\ time\ i$$

$$\hat{y}_i - predicted\ value\ of\ y$$

Even though all the above measures were calculated for all the models, the MAPE metric was the selected one to make the comparison between the models, since it was verified in the Literature Review of this study to be the most used. The forecast presenting the smallest MAPE is the most accurate among the others.

According to Lewis & C.D.(1982) if the value of MAPE is inferior to 10% then we can classify the forecast as very good, if it is between 11% and 20% it is a good forecast, if it is between 21% and 50% then it is a reasonable forecast, and if it is above 50%, it indicates an inaccurate forecast.

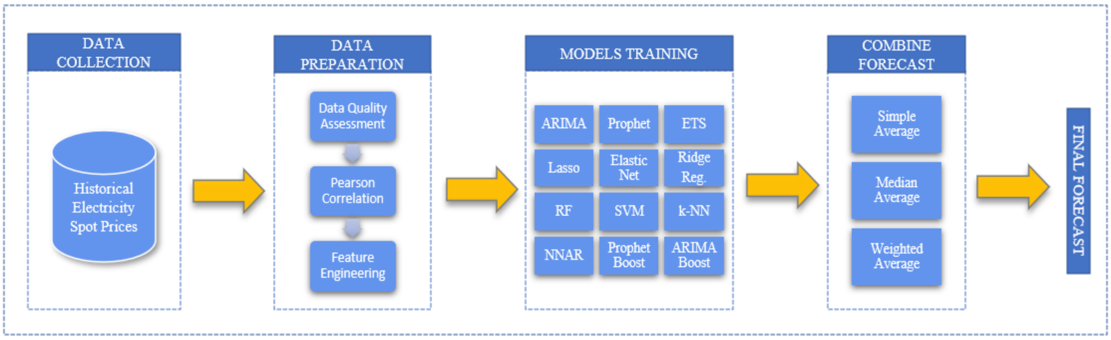
2.2.4 Models Combination

After the twelve models were created and the parameters were tuned, the respective predictions were obtained and the accuracy calculated, the models were combined, using an "ensemble" approach (Vannitsem, Wilks, & Messner, 2018). The main advantages of combining models' predictions is achieving a lower error and less overfitting. The top 4 models with best accuracy were identified and chosen to be combined. The same process was repeated using only the top 3 and top 2 models. The performance results were compared. The models were combined, by averaging their predictions, using three different approaches: a simple average (mean), median average, and a weighted average.

- **Simple Average:** Weighs all models with the same proportion.
- **Median Average:** No weighting. Selects prediction using the centered value for each time stamp.
- **Weighted Average:** The weights are manually selected and adjusted according to the performance of each specific model. The weights are assigned such that the sum of weights must be equal to 1.

Figure 29 summarizes all the process described in the Data and Methodology chapter.

Figure 29 Scheme diagram of the process leading to the final forecast



Source: Own Work.

3 RESULTS AND DISCUSSION

This chapter contains the results and discussion of the two implemented approaches: the monthly forecast (uses monthly prices data) and the daily forecast (uses daily prices data).

3.1 Monthly Forecast

The monthly approach envisages to predict 12 points in the future corresponding to the ESP of each month of the next year.

3.1.1 Test Set Accuracy

The detailed error measurements results of each parameter combination, in the grid search of each model using monthly data, can be seen in the **Appendix.** Table 6 summarizes the parameters with the lowest MAPE by model. In this phase, we compared twelve optimized models. Prophet is the model with the higher error in its prediction (20.02%), while ARIMA Boost has the lowest error (12.39%). Since the MAPE of all the models, using monthly data, lies between 10% and 20%, they can be classified as good forecasts (Lewis & C.D., 1982).

Table 6 Accuracy Table – Monthly approach

Model	Parameters	Model Type	MAE	MAPE	MASE	SMAPE	RMSE
ARIMA	AUTO: ARIMA(2,0,1)	Statistical	8.08	14.68	1.73	15.52	10.03
Prophet	AUTO	Statistical	11.01	20.02	2.36	22.50	12.81
Exponential Smoothing	AUTO: ETS(A,N,N)	Statistical	7.84	14.38	1.68	15.03	9.75
LASSO	- Penalty: 3 - Mixture: 1	Statistical	8.18	15.48	1.75	15.71	9.63
Elastic Net	- Penalty: 7 - Mixture: 0.5	Statistical	8.51	15.45	1.82	16.42	10.38
Ridge Regression	- Penalty: 5000 - Mixture: 0	Statistical	8.65	15.50	1.85	16.70	10.70
Random Forest	- Mtry (F): 15 - Trees (K): 40 - Min Node: 5	Computational Intelligence	8.58	15.82	1.84	16.61	10.11
Support Vector Machine	- Cost: 0.001 - Sigma: 0.001	Computational Intelligence	8.48	15.23	1.82	16.34	10.53
k-Nearest Neighbor	- Neighbors: 2 - Distance: 1	Computational Intelligence	7.84	15.07	1.68	15.01	10.77
Neural Network Autoregression	- p: 1 and P: 0 - K: 2 - Epochs: 300	Computational Intelligence	6.42	13.06	1.38	12.41	7.68
Prophet Boost	AUTO	Hybrid	7.71	14.97	1.65	15.15	9.11
ARIMA Boost	AUTO	Hybrid	6.84	12.39	1.47	13.04	8.62

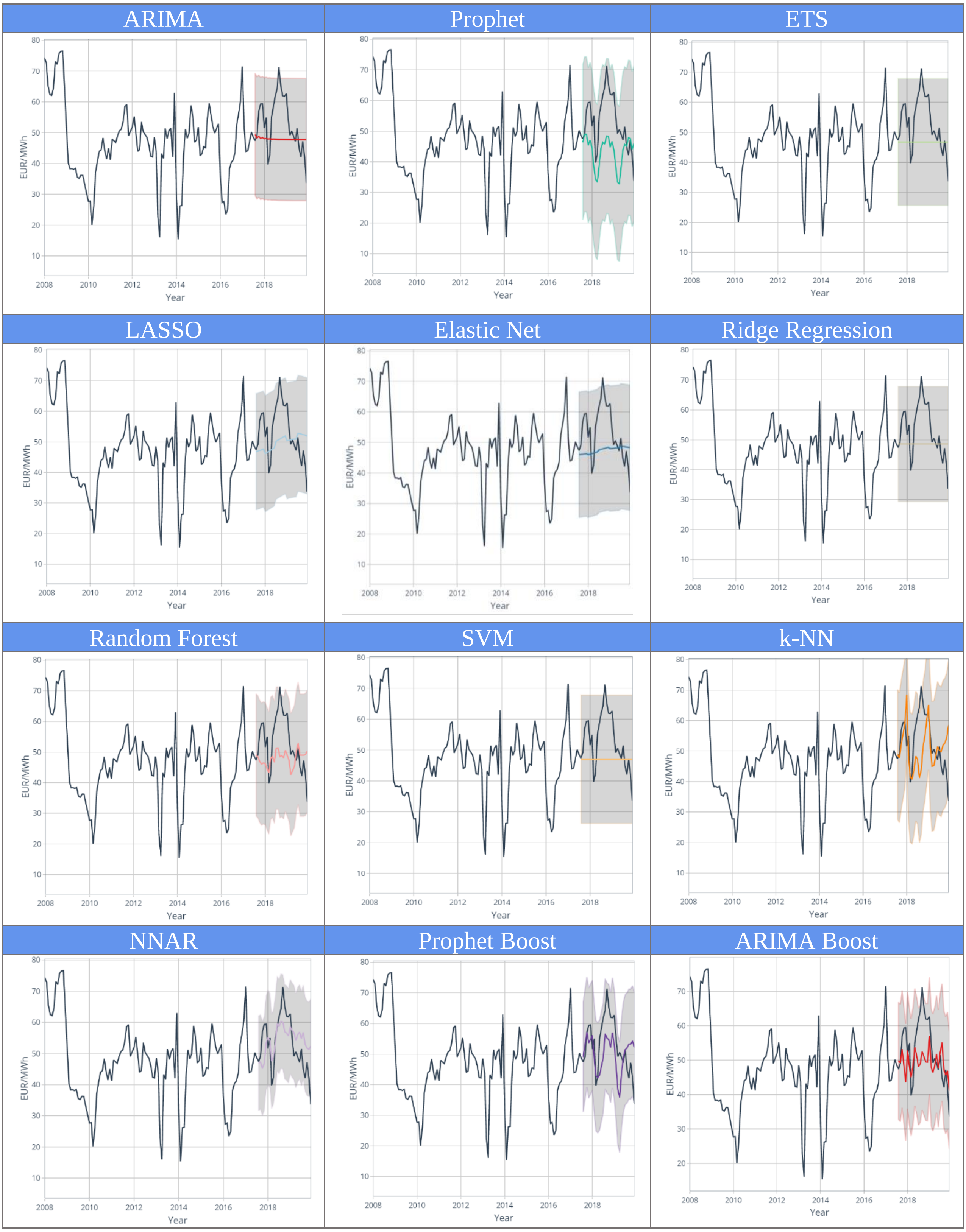
Source: Own Work.

3.1.2 Test Set Forecast

It is extremely important to plot the forecasts of the test set, because just analyzing the accuracy of the model is not enough to understand if the model predictions are in fact good or not. The results of the output forecast of each optimized model presented in Table 6 are plotted below in Figure 30. The original time-series is presented in dark grey and the forecast of the test data is represented with a different color for each model. The light grey is the interval of confidence of each forecast model. In all the models, the real values are inside that interval, which can be translated to a good accuracy. We can see that ARIMA, ETS, LASSO, Elastic Net, Ridge Regression and SVM present a minimal fluctuation and trend, or none at all, having a behavior close to a straight line. In contrast, Prophet, Random Forest, k-NN, NNAR, ARIMA Bost and Prophet Bost consider a fluctuation and trend in the price's forecast, being closer to the expected values behavior. We can conclude that the

Computational Intelligence (except the SVM model) and *Hybrid* models, tend to be sensitive to the fluctuation of the prices, in opposition to the *Statistical* models, in the case of a monthly forecast approach.

Figure 30 Plot of each model test set forecast - Monthly approach



Source: Own Work.

3.1.3 12 month-ahead forecast

Given that ARIMA, ETS, LASSO, Elastic Net, Ridge Regression and SVM showed a poor forecast in Figure 30, compared to the remaining, those were not used in the 12 month-ahead forecast. The rest of the models were implemented, using the historical data from 2008 to 2019 to monthly forecast the year of 2020. The plotted results of each model are represented in Figure 31.

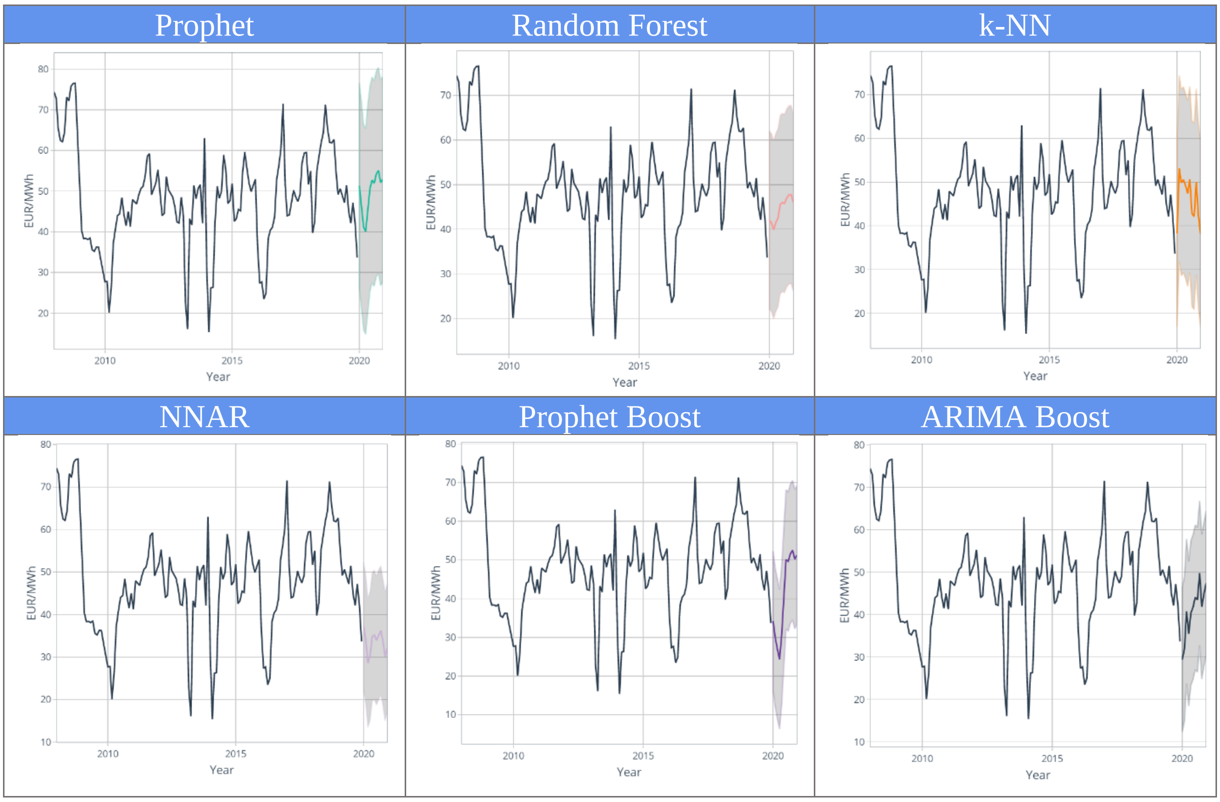


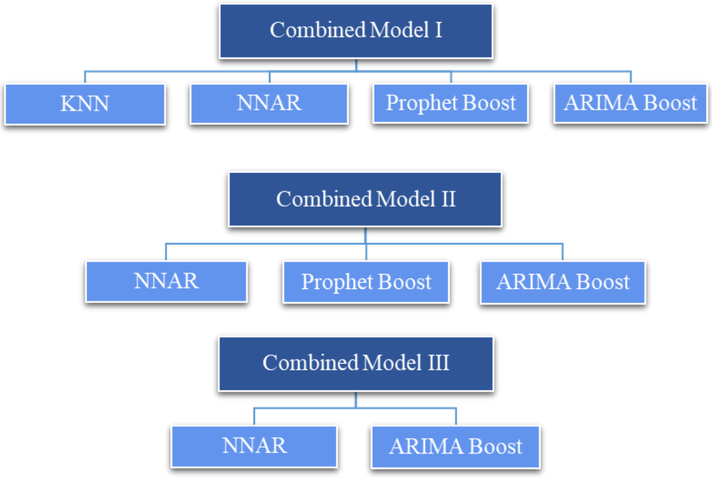
Figure 31 Plot of each model 12 month-ahead forecast

Source: Own Work.

3.1.4 Combined Models

From the 6 models with the best forecast (Prophet, Random Forest, k-NN, NNAR, ARIMA Boost and Prophet Boost), the top 4 with lowest MAPE error (k-NN, NNAR, ARIMA Boost and Prophet Boost, see Table 6) were selected as input for the creation of the *simple average* model combination (or ensemble) depicted in Figure 32. In the first combined model, these top 4 models were selected. In the second combined model, the top 3 models were selected, therefore k-NN was excluded. In the third one, the top 2 models with better MAPE performance were selected: NNAR and ARIMA Boost. Then, the process was repeated but using *median average*. In total, 6 models were created.

Figure 32 Diagram of the combined models - Monthly approach



Source: Own Work.

For the weighted average model, only the top 2 (NNAR+ ARIMA Boost) were used as input for the combination. Therefore, the weights of these models were changed empirically in order to find an optimized result. The weight combination with the lowest error was: w1=0.28 for NNAR and w2=0.72 for ARIMA Boost.

Table 7 lists the results of the error calculations of each of the 6 combined models and the weighted model.

Table 7 Accuracy table of the combined models – Monthly approach

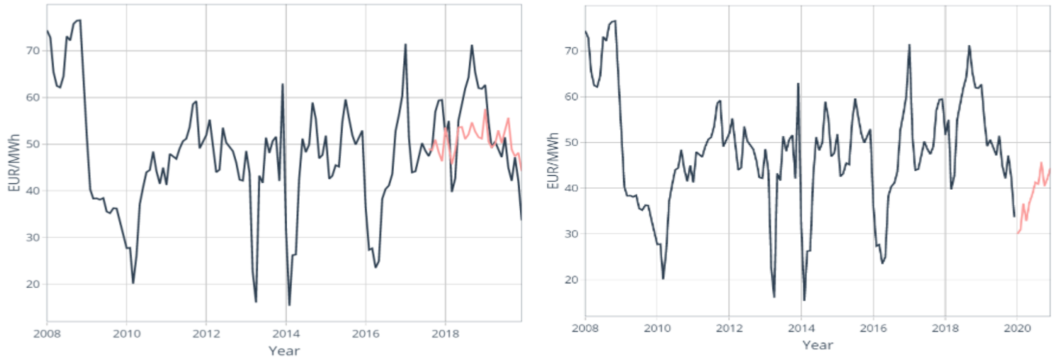
Model	MAE	MAPE	MASE	SMAPE	RMSE
4 models (MEAN)	6.65	12.77	1.43	12.69	7.99
4 models (MEDIAN)	6.79	13.13	1.46	13.00	8.20
3 models (MEAN)	6.42	12.34	1.38	12.28	7.46
3 models (MEDIAN)	6.63	12.94	1.42	12.70	7.98
2 models (MEAN)	6.26	11.99	1.34	11.96	7.46
2 models (MEDIAN)	6.26	11.99	1.34	11.96	7.46
2 models (WEIGHTED)	6.36	11.84	1.36	12.09	7.82

Source: Own Work.

The models tend to have a lower error when the number of models combined is smaller. The ensemble model with the lowest error found was the 2 models weighted combination (NNAR+ARIMA Boost weighted model) with a MAPE = 11.84%. This model is not only better than any combination tested, but is also better than any of the 12 single models, as can be seen in Table 6 and Table 7.

In Figure 33 (a) we plot the price's forecast of the test data using the NNAR+ARIMA Boost combined weighted model and in Figure 33Figure 33 (b) we plot the 12 month-ahead forecast of the prices using the NNAR+ARIMA Boost combined weighted model.

Figure 33 Plot of the test set forecast (a) and 12 month-ahead forecast (b)



(a) *(b)*

Source: Own Work.

3.2 Daily Forecast

The daily forecast approach envisages to estimate 365 points in the future corresponding to the ESP of each day of the next year.

3.2.1 Test Set Accuracy

Much like for the monthly case, the error measurements results of each parameter combination in each grid search of each model using daily data can be seen in the **Appendix.** Table 8 summarizes the tunned parameters with the lowest MAPE for each model. As expected, the error is much higher when using daily data, since the granularity of the data and the estimated points are larger, when compared to the monthly approach. Prophet Boost is the model with the highest error in its prediction (27.71%), while Exponential Smoothing has the lowest error (24.37%). Since the MAPE of all the models, using daily data, lies between 24% and 28%, they can be classified as close to good reasonable forecasts (Lewis & C.D., 1982).

Table 8 Accuracy Table – Daily approach

Model	Parameters	Model Type	MAE	MAPE	MASE	SMAPE	RMSE
ARIMA	AUTO: ARIMA(4,1,1)(2,0,0)[7]	Statistical	9.43	25.24	2.48	18.78	11.94
Prophet	AUTO	Statistical	9.12	26.06	2.40	18.26	11.09

(table continues)

(continued)

Model	Parameters	Model Type	MAE	MAPE	MASE	SMAPE	RMSE
Exponential Smoothing	AUTO: ETS(A,N,A)	Statistical	9.19	24.37	2.42	18.39	11.62
LASSO	- Penalty: 3 - Mixture: 1	Statistical	9.77	25.48	2.57	19.53	12.10
Elastic Net	- Penalty: 6 - Mixture: 0.5	Statistical	9.78	25.47	2.57	19.53	12.14
Ridge Regression	- Penalty: 5000 - Mixture: 0	Statistical	9.89	25.67	2.60	19.74	12.42
Random Forest	- Mtry (F): 21 - Trees (K): 500 - Min Node: 5	Computational Intelligence	8.57	25.00	2.25	16.95	11.25
Support Vector Machine	- Cost: 1 - Sigma: 0.1	Computational Intelligence	8.44	24.38	2.22	16.84	10.85
k-Nearest Neighbor	- Neighbors: 5 - Distance Type: 1	Computational Intelligence	9.03	25.95	2.37	18.18	11.60
Neural Network Autoregression	- p: 11 - P 10 - K: 11 - Epochs: 100	Computational Intelligence	8.01	24.58	2.11	15.92	10.51
Prophet Boost	AUTO	Hybrid	10.04	27.71	2.64	19.95	12.17
ARIMA Boost	AUTO	Hybrid	9.52	24.92	2.50	19.04	11.95

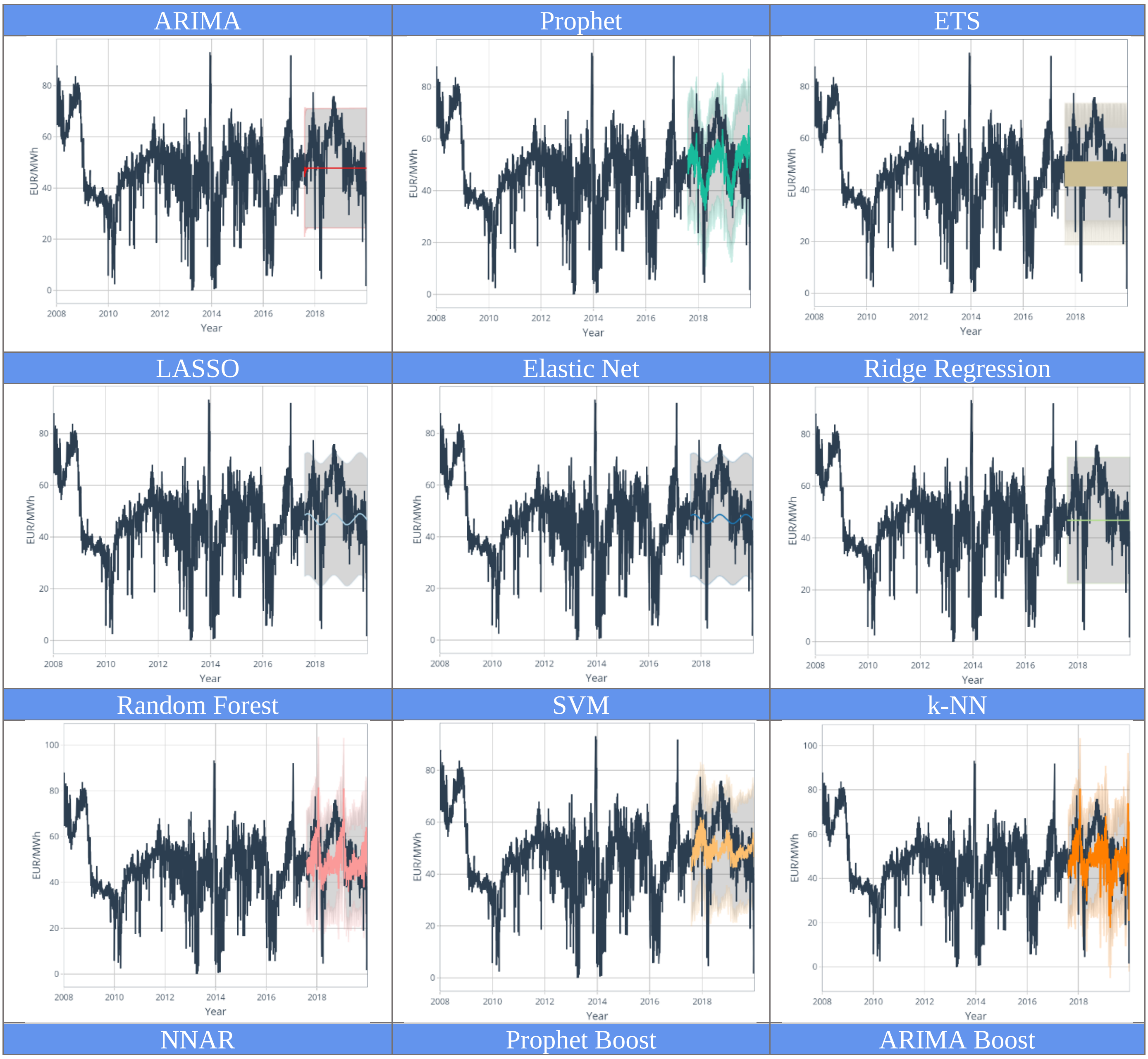
Source: Own Work.

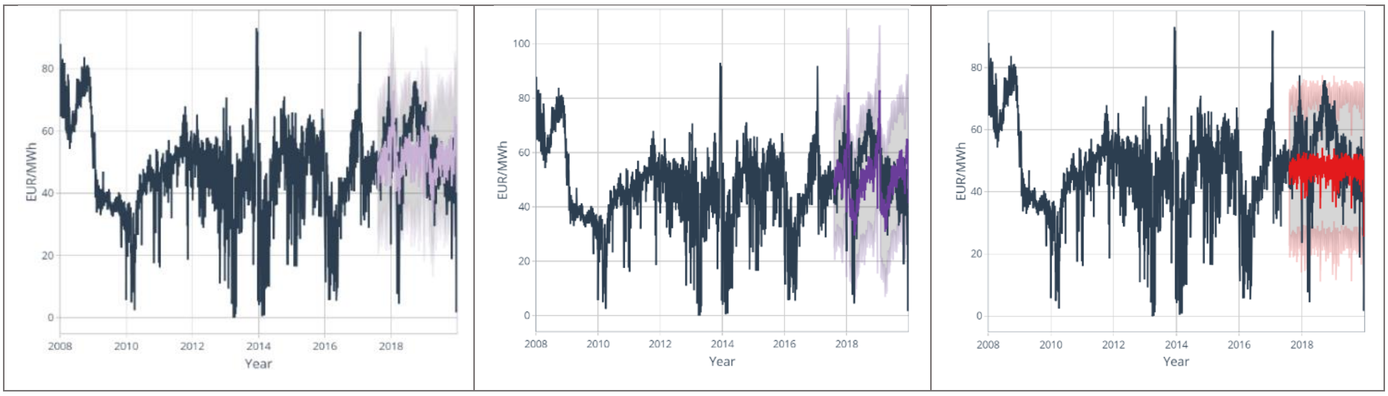
3.2.2 Test Set Forecast

Like for the monthly case, it is very relevant to plot the daily forecasts of the test set, because just analyzing the accuracy of the model is not enough to understand if each model forecast is in fact good or not. The results of the output forecast of each optimized model presented in Table 8 are depicted below in Figure 34. The original time-series is presented in dark grey and the forecast of the test data is represented with a different color for each model. The light grey is the interval of confidence of each model forecast. In all the models, there are some real values that are outside that interval. These values are mostly spikes that the models failed to identify. We can see that ARIMA, ETS, LASSO, Elastic Net, and Ridge Regression present a minimal fluctuation and trend, or none at all, having a behavior close to a straight line. In contrast, Prophet, Random Forest, SVM, k-NN, NNAR, and ARIMA Bost, Prophet Bost consider a fluctuation and trend in the forecasted prices, being closer to the expected values behavior. So, the *Computational Intelligence* and *Hybrid* models tend to be sensitive to the fluctuation of the prices, as opposed to the *Statistical* models, in a daily forecast approach.

By comparing the results depicted in Table 6 and Table 8, the models that reflected better forecast in the monthly approach are the same ones in the daily approach. The only exception was SVM. This model presented a forecast in the range of the best ones in the daily approach, and a poor one in the monthly approach.

Figure 34 Plot of each model test set forecast - Daily approach



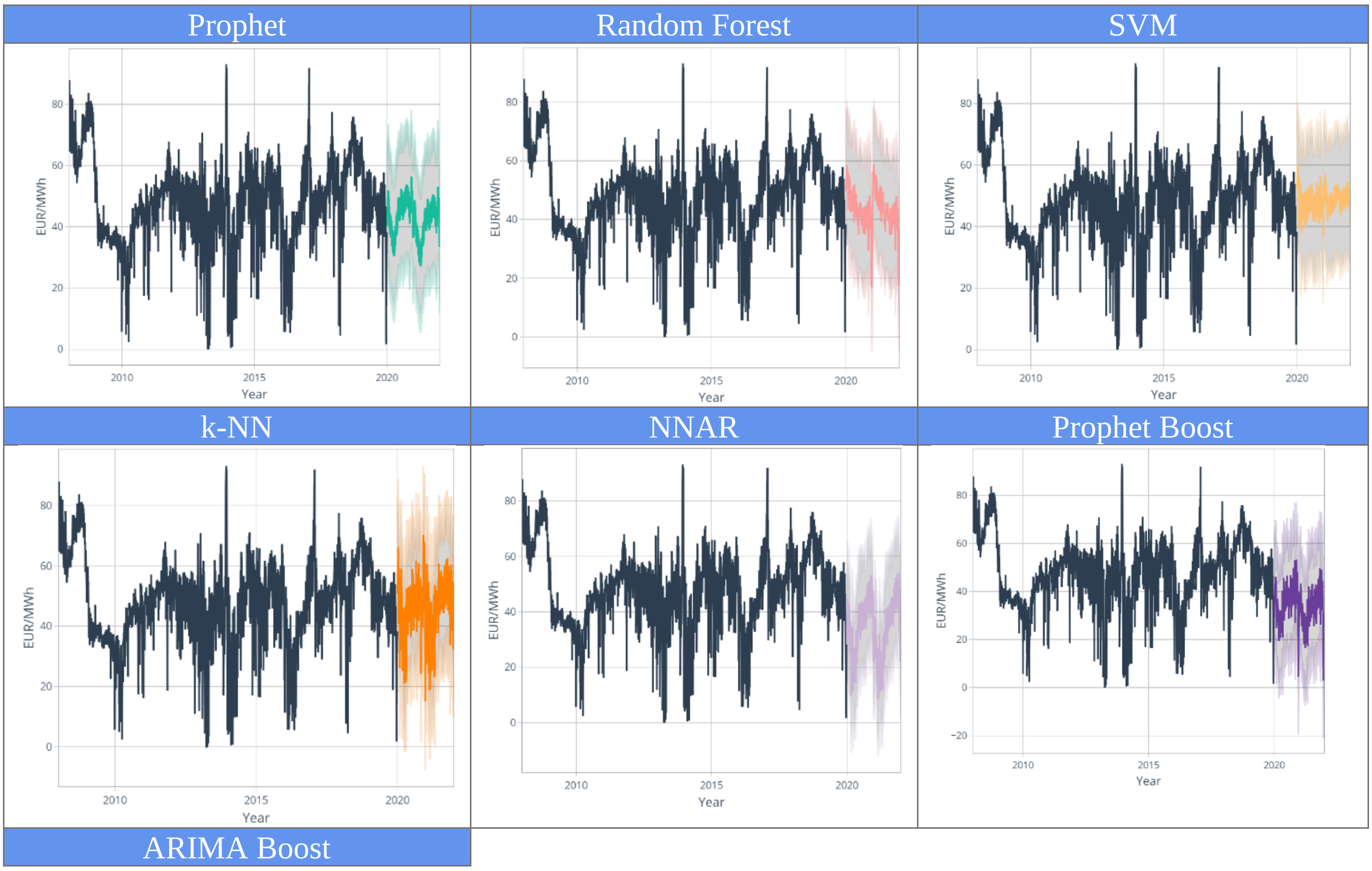


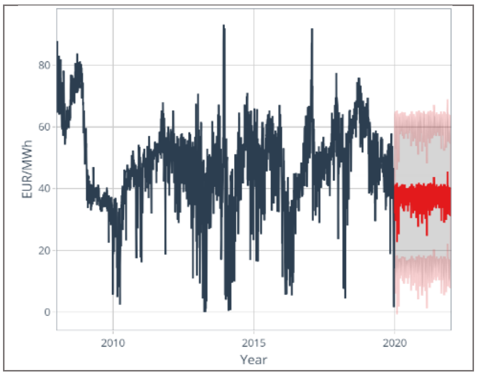
Source: Own Work.

3.2.3 365 day-ahead forecast

Since ARIMA, ETS, LASSO, Elastic Net, and Ridge Regression showed a poor forecast in Figure 34, those were not used in the 365 day-ahead forecast. The rest of the models were implemented, using the historical data from 2008 to 2019 to daily forecast the year of 2020. The plotted results of each model are represented in Figure 35.

Figure 35 Plot of each model 365 day-ahead forecast



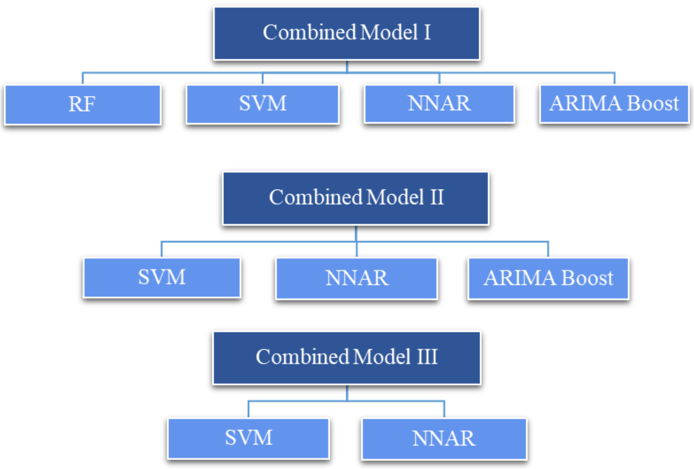


Source: Own Work.

3.2.4 Combined Models

From the 7 models with the best forecast (Prophet, Random Forest, SVM, k-NN, NNAR, ARIMA Boost and Prophet Boost), the top 4 with lowest MAPE error (Random Forest, SVM, NNAR, and ARIMA Boost) were selected as input for the creation of the *simple average* model combination (Figure 36). In the first model, these top 4 models were selected. In the second combined model, the top 3 models were selected, therefore Random Forest was excluded. In the third one, the top 2 models with better MAPE performance were selected: NNAR and SVM. Subsequently, the process was repeated but using *median average*. In total, 6 models were created.

Figure 36 Diagram of the combined models - Daily approach



Source: Own Work.

For the weighted average model, only the top 2 (NNAR+SVM) were used as input for the combination. Therefore, the weights of these models were changed in order to find an optimized result. Although, a lowest error was not found. Therefore, the weight combination with the lowest error was: w1 = 0.5 for NNAR and w2 = 0.5 for SVM. Table 9 shows the results of the error calculations of each of the 6 models and the weighted model.

Table 9 Accuracy table of the combined models – Daily approach

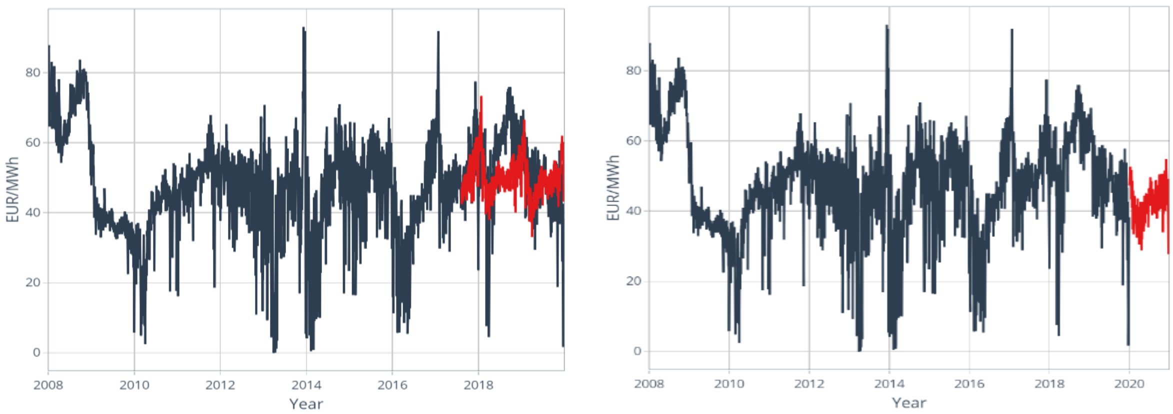
Model	MAE	MAPE	MASE	SMAPE	RMSE
4 models (MEAN)	8.19	23.88	2.15	16.32	10.55
4 models (MEDIAN)	8.28	24.18	2.18	16.50	10.68
3 models (MEAN)	8.29	23.91	2.18	16.52	10.64
3 models (MEDIAN)	8.35	24.19	2.19	16.63	10.77
2 models (MEAN)	8.01	24.07	2.11	15.96	10.42
2 models (MEDIAN)	8.01	24.07	2.11	15.96	10.42
2 models (WEIGHTED)	8.01	24.07	2.11	15.96	10.42

Source: Own Work.

Contrary to what resulted in our monthly data approach, the models in the daily data case, tend to have a higher error when the number of models combined is lower. The model with the lowest error found was the 4 models mean combination, with a MAPE = 23.88%. According to our results, this model is not only better than any combination tested, but also better than any of the 12 single models.

In (a) we plot the forecast of the test set using the RF+SVM+NNAR+ARIMA Boost mean combined model and Figure 37 (b) is plotted the 365 day-ahead forecast of the prices using the RF+SVM+NNAR+ARIMA Boost mean combined model.

Figure 37 Plot of the test set forecast (a) and 365 day-ahead forecast (b)



(a) *(b)*

Source: Own Work.

CONCLUSION

A mid-term forecast of the electricity spot prices is a complex task, where the predicted results can sometimes be dubious, due to the volatile and complex electricity market conditions. However, such forecast is essential for every market participant and for investment decision making in electricity markets, helping to reduce the risk of their investments, since these predictions give new information about the behavior of the ESP in the future. Furthermore, these types of forecasts are very useful for the consumers that have their tariffs indexed to the electricity spot prices, to know when to increase or decrease their electricity consumption and, also, for the energy storage systems, helping these to determine when to store or distribute the energy.

This study deals with an ESP mid-term forecast problem, attempting to predict the daily and monthly ESP for the next year-ahead, using monthly and daily historical data that ranges from 2008 till 2019 (12 years). The first part of this study consisted of analyzing the characteristics of the electricity as well as the structure of the electricity market, with special focus on the Iberian Market. Then the possible forecast techniques and approaches were explained. Subsequently, a Systematic Review and Meta-analysis of the Literature of the last 5 years, related to the mid-term ESPF was implemented, using the PRISMA methodology. The number of papers identified was sparse. Only thirteen papers were found, from which only five implemented computation intelligence models. Furthermore, from the thirteen papers, only four used monthly resolution data and none used daily data. Also, regarding the deterministic approach, only one of the 13 papers forecasted up to one-year horizon. There is therefore, a large gap in the literature when it comes to mid-term approaches for ESPF.

In this study, the Portuguese ESP data was explored and analyzed. The data showed high volatility and fluctuation, and a weekly and yearly seasonality was identified. After that, different forecasting models were implemented. The chosen models were: Statistical models (ARIMA, Exponential Smoothing, LASSO Regression, Ridge Regression, Elastic Net Regression, and Prophet), Computational Intelligence models (NNAR, RF, SVM, and k-NN) and hybrid models (ARIMA Boost and Prophet Boost). This study compares the forecasting performance derived from these twelve individual and hybrid models, as well as from 7 combinations between those models.

Discussion of the Research Hypotheses

After testing and comparing the models, when forecasting mid-term ESP in the day-ahead OMIE market, the results of the RH were verified and summarized in Table 10.

Table 10 Summary of hypotheses' results.

Nr.	Hypothesis	Result
RH1	Computational Intelligent models show better performance if compared to Statistical models, when forecasting mid-term ESP in the day-ahead OMIE market.	Confirmed

RH2	The forecast of mid-term ESP in the day-ahead OMIE market, using Computational Intelligent or Statistical models, has better performance when using monthly price data than daily price data.	Confirmed
RH3	It is possible to improve the forecast of mid-term ESP in the day-ahead OMIE market, by selecting an ensemble of models, either trained with monthly or daily price data.	Confirmed

Source: Own Work.

The ***RH1*** was proved to be true based on the results of this thesis, since Computational Intelligence models tend to outperform statistical models, evidencing more accurate results when the forecast data is compared to the expected values, by calculating the accuracy measures and plotting both time-series. The ***RH2*** was also accepted vis-a-vis the results of this thesis. Forecasting the next year with monthly data has a lower error compared to when forecasting with daily data. All the 12 models had a lower MAPE (between 12% and 20%) when using monthly data compared to when using daily data (between 24% and 28%). The ***RH3*** was also confirmed by the results of this thesis. The best models using monthly or daily data are NNAR and ARIMA Boost. However, by combining models' forecasts, either when using daily or monthly data, we achieved a higher accuracy of the forecast, when compared to single models. For a monthly approach, the combination of **NNAR+ARIMA Boost,** using a weighted average, outperformed any other combination or individual model with a MAPE=11.84%. Regarding the daily approach, it was the combination of **RF+SVM+NNAR+ARIMA Boost,** by applying a mean average, that outperformed any other combination or individual model with a MAPE=24.07%. Therefore, we can conclude that the combination of models provides a more accurate and reliable prediction, enhancing the chances of capturing the behavior of the ESP in the future, compared with the application of single time series forecasting methods.

Contributions of this study

This study helps advance the field of forecasting ESP in a mid-term horizon, helping to fill the gap of mid-term ESPF focused literature studies, doing things that were never done and seen before in the literature, to the best of the authors' knowledge, like:

- Implementing and comparing different models that were never tested in a mid-term horizon.
- Implementing and comparing different models that were tested using Portuguese ESP as input.
- Introducing a new ensemble method that consists in combining individual models to forecast ESP, resulting in a better accuracy.
- Introducing a daily approach for forecasting ESP in a mid-term horizon, using daily data as input to the models, to forecast the next 365 days-ahead.
- Comparing two different data resolution approaches (monthly and daily) instead of focusing in just one.
- Comparing different type of models (statistical, computational intelligence and hybrid models) in the same study, instead of focusing in just one.

Limitations and difficulties in this study

Since forecasting ESP in a mid-term horizon is a complex problem, different obstacles were encountered like:

- Forecasting the next entire year with hourly data. The dimension of the data and hourly fluctuation is too high to be reliably forecasted.
- Using external factors for the forecast like weather variables, consumption, and production variables. The forecast of these variable in one-year horizon is an extremely complex and much needed task, that goes out of the scope of this study.
- The incapability of the models to extrapolate the forecasts. The accuracy decreases with the increase of the forecast horizon window size, since uncertainty and bias tend to grow over time. In a mid-term horizon, forecasting one-year ahead is more challenging than just weeks or few months ahead.
- Tuning of many parameters. There are no standardized methods for determining the optimal value of the parameters for each model. In practice they are chosen by trial-and-error and in some models the specific prediction process cannot be explained.
- This study presented techniques, models, and approaches only using historical ESP from Portugal. Other countries' prices may present different characteristics and behaviors, requiring different approaches. So, the methods and models of this study must be changed and adjusted in order to be implemented in other markets.
- The limitation of the local computer capacities to run the models.

Suggestions for future research

There is still room for improvement, and the following tasks can be implemented in future works:

- Use the same models and techniques but with data from other countries and electricity markets, to test and compare the accuracy of the models.
- Implement a weekly averaged price one-year forecast and compare the performance of the models.
- Implement more promising models, for example the LSTM model, by leveraging high performance computing resources in the cloud.
- Test and compare more combinations of parameters in some models and automate this process.
- Develop better schemes to further improve the selection of the models' parameters.
- Create more hybrid models.
- Combine more models and compare their performance.
- Study and try to create reliable forecasts for weather, load, and generation variables for the next year in order to use them as input to forecast the ESP, creating multi-dimensional data sets to train and test the models.
- Study and implement techniques that help to forecast the spikes in the ESP during the following year.

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APPENDICES

Appendix 1: Povzetek (Summary in Slovene language)

Vmesna napoved električnih cen je precej kompleksna naloga, ne samo, da je električna energija edinstveno blago, ampak imajo takšni trgi nekatere posebne in edinstvene značilnosti. Je bistvenega pomena za vsakega udeleženca na trgu z električno energijo, za vse namene naložb, odločanja, porabe, distrubucije, razporejanja in načrtovanja strategije ponudb.

Primarni namen te disertacije je uvesti in primerjati računalniško inteligenco, statistične in hibridne modele za napovedovanje dnevnih in mesečnih portugalskih cen električne energije, za dan vnaprej, z obdobjem enega leta. Modeli, uporabljeni v tej disertaciji, so: *Autoregressive Neural Network (NNAR), Support Vector Machine (SVM), K-Nearest Neighbor (k-NN), Random Forest (RF), Autoregressive Integrated Moving Average (ARIMA), Exponential Smoothing (ETS), Elastic Net, LASSO Regression, Ridge Regression, Prophet , ARIMA + Extreme Gradient Boosting (XGBoost), and Prophet + Extreme Gradient Boosting (XGBoost)*. V magistrskem delu je prikazana napoved uspešnosti dvanajstih posameznih in hibridnih modelov, ter tudi iz sedmih kombinacij med temi modeli.

Ugotovili smo, da računalniški inteligenčni modeli običajno presegajo statistične. Tudi uporaba mesečnih podatkov kot vhodnih podatkov za modele, zmanjšuje natančnost napak pri rezultatih v primerjavi z uporabo dnevnih podatkov. Ključna ugotovitev nam pove, da je z združevanjem napovedi modelov z uporabo dnevnih ali mesečnih podatkov dosežena večja natačnost napovedi kot v primerjavi s posameznimi modeli.

Appendix 2: Summary in English language

A mid-term forecast of the electricity spot prices is a complex task, not only electricity is a very singular commodity, but also the electricity market has some specific and unique characteristics. However, it is essential for every electricity market participant, for electricity market investment, decision making, consumption, distribution, scheduling, and bidding strategy planning purposes.

The primary purpose of this dissertation is to implement and compare Computational Intelligence, statistical, and hybrid models to forecast the daily and monthly Portuguese electricity day-ahead spot prices, with a one-year horizon period. The models used in this dissertation are: Autoregressive Neural Network (NNAR), Support Vector Machine (SVM), K-Nearest Neighbor (k-NN), Random Forest (RF), Autoregressive Integrated Moving Average (ARIMA), Exponential Smoothing (ETS), Elastic Net, LASSO Regression, Ridge Regression, Prophet , ARIMA + Extreme Gradient Boosting (XGBoost), and Prophet + Extreme Gradient Boosting (XGBoost). This thesis compares the forecasting performance derived from these twelve individual and hybrid models, as well as from 7 combinations between those models.

We found out that computational intelligence models tend to outperform statistical ones. Also, using monthly data as input for the models lowers the error accuracy of the results, compared to when using daily data. Finally, by combining models' forecasts, either when using daily or monthly data, a higher accuracy of the forecast was achieved, when compared to single models.

Appendix 3: Acknowledgments

I wish to express my deepest regards and thanks to Prof. Dr. José Miguel Dias for his supervision, motivation, and support during the entire process of my dissertation. Also, to Prof. Dr. Aleš Popovič and Prof. Dr. Vitor Duarte dos Santos. Without their experience and guidance, this work would not be possible.

Additionally, many thanks to the assistance and support, given in this dissertation, by Tiago Silva Pedro, Maria Anastasiadou, and Ahmed Abdelaziz.

Finally, I want to thank to my beloved family and friends, especially to my mother, my father, and both my grandmothers. Their constant support, dedication and encouragement lead me to where I am today.

This dissertation, as well all my five student academic years, are dedicated to my grandfather that unfortunately left me during this last phase of my student journey, unable to see me conclude it. I hope he is proud.

"If I have seen further it is by standing on the shoulders of Giants"

\- Isaac Newton

Appendix 4: Tables and plots of the results of the parameters tuning

Refer to chapter *2.2.2 Parameters Tuning*. Source: Own work.

LASSO – Monthly Data

MIXTURE	*PENALTY*	*MAE*	*MAPE*
1	0.01	53.63	107.44
1	0.5	25.21	52.13
1	1	15.64	33.41
1	2	9.23	19.88
1	3	8.18	15.48
1	4	8.63	15.54
1	5	8.65	15.50
1	6	8.65	15.50
1	7	8.65	15.50
1	8	8.65	15.50
1	9	8.65	15.50
1	10	8.65	15.50
1	20	8.65	15.50
1	30	8.65	15.50
1	40	8.65	15.50
1	50	8.65	15.50
1	100	8.65	15.50

Elastic Net – Monthly Data

MIXTURE	*PENALTY*	*MAE*	*MAPE*
0.5	0.01	53.18	106.28
0.5	0.5	28.16	57.81
0.5	1	20.92	43.70
0.5	2	13.27	28.68
0.5	3	9.54	20.80
0.5	4	8.66	17.96
0.5	5	8.30	16.24
0.5	6	8.15	15.50
0.5	7	8.51	15.45
0.5	8	8.62	15.50
0.5	9	8.63	15.47
0.5	10	8.65	15.50
0.5	20	8.65	15.50
0.5	30	8.65	15.50
0.5	40	8.65	15.50
0.5	50	8.65	15.50
0.5	100	8.65	15.50

Ridge Regression – Monthly Data

MIXTURE	*PENALTY*	*MAE*	*MAPE*
0	0.01	29.28	59.79
0	0.5	28.75	58.75
0	1	22.41	46.34
0	2	15.69	33.11
0	3	12.35	26.43
0	4	10.64	22.85
0	5	9.84	20.95
0	6	9.38	19.75
0	7	9.27	19.21
0	8	9.22	18.88
0	9	9.28	18.76
0	10	9.34	18.70
0	20	9.67	18.37
0	30	9.61	17.89
0	40	9.61	17.89
0	50	9.46	17.31
0	100	9.25	16.72
0	200	9.03	16.24
0	1000	8.74	15.67
0	2000	8.7	15.59
0	4000	8.67	15.54
0	5000	8.65	15.5
0	6000	8.65	15.5
0	7000	8.65	15.5

Lasso – Daily Data

MIXTURE	*PENALTY*	*MAE*	*MAPE*
1	0.01	12.35	28.11
1	0.5	11.77	27.44
1	1	11.21	26.84
1	2	10.21	25.91
1	3	9.77	25.48
1	4	9.83	25.55
1	5	9.89	25.67
1	6	9.89	25.67
1	7	9.89	25.67
1	8	9.89	25.67
1	9	9.89	25.67
1	10	9.89	25.67
1	20	9.89	25.67
1	30	9.89	25.67
1	40	9.89	25.67
1	50	9.89	25.67
1	100	9.89	25.67

Elastic Net – Daily Data

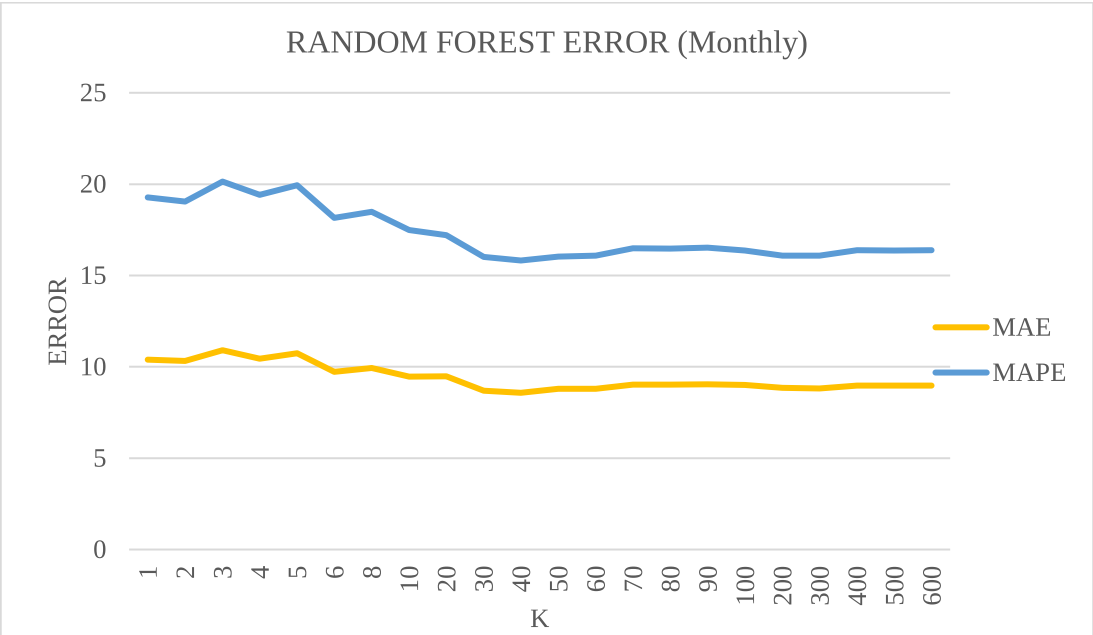
MIXTURE	***PENALTY***	***MAE***	***MAPE***
0.5	0.01	12.36	28.14
0.5	0.5	12.03	27.73
0.5	1	11.74	27.36
0.5	2	11.16	26.75
0.5	3	10.63	26.23
0.5	4	10.19	25.86
0.5	5	9.81	25.53
0.5	6	9.78	25.47
0.5	7	9.80	25.51
0.5	8	9.84	25.58
0.5	9	9.89	25.67
0.5	10	9.89	25.67
0.5	20	9.89	25.67
0.5	30	9.89	25.67
0.5	40	9.89	25.67
0.5	50	9.89	25.67
0.5	100	9.89	25.67

Ridge Regression – Daily Data

MIXTURE	***PENALTY***	***MAE***	***MAPE***
0	0.01	12.31	28.10
0	0.5	12.31	28.10
0	1	12.28	28.11
0	2	12.22	28.11
0	3	12.16	28.08
0	4	12.11	28.05
0	5	12.06	28.02
0	6	12.00	27.98
0	7	11.96	27.94
0	8	11.91	27.90
0	9	11.86	27.86
0	10	11.81	27.82
0	20	11.46	27.48
0	30	11.20	27.21
0	40	11.01	27.01
0	50	10.87	26.85
0	100	10.48	26.42
0	200	10.2	26.07
0	1000	9.94	25.74
0	2000	9.92	25.7
0	4000	9.9	25.68
0	5000	9.89	25.67
0	6000	9.89	25.67
0	7000	9.89	25.67

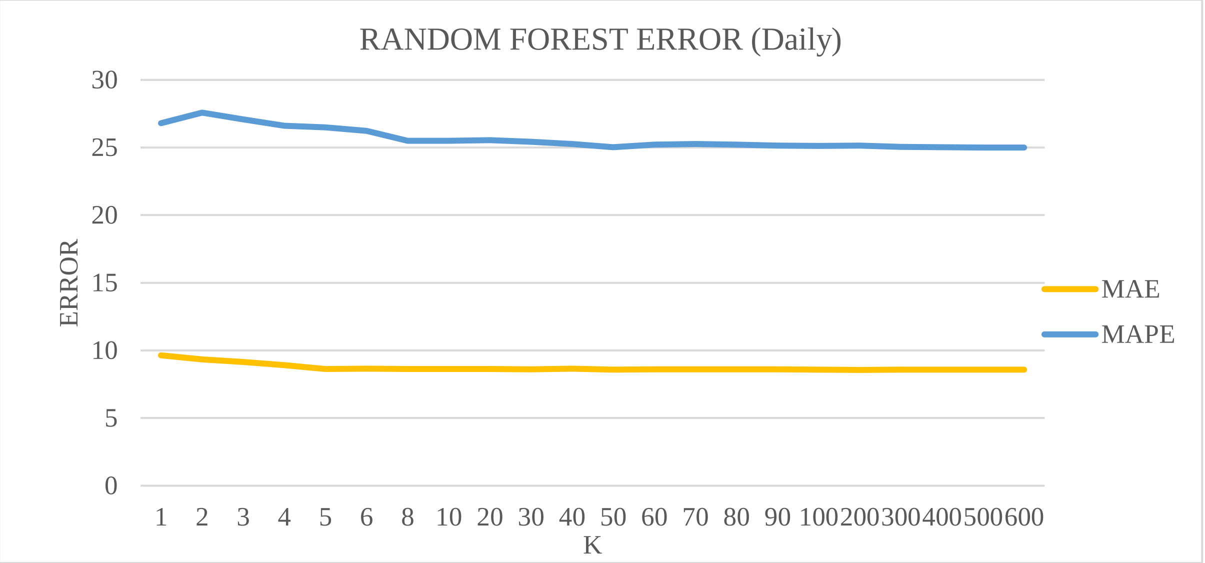
Random Forest – Monthly Data

K	***MAE***	***MAPE***
1	10.4	19.28
2	10.33	19.05
3	10.91	20.14
4	10.45	19.41
5	10.74	19.94
6	9.73	18.16
8	9.94	18.49
10	9.47	17.48
20	9.49	17.20
30	8.70	16.01
40	8.58	15.82
50	8.80	16.04
60	8.80	16.08
70	9.02	16.49
80	9.03	16.47
90	9.04	16.53
100	9.01	16.37
200	8.85	16.09
300	8.81	16.08
400	8.97	16.38
500	8.97	16.36
600	8.98	16.39



Random Forest – Daily Data

K	*MAE*	*MAPE*
1	9.64	26.80
2	9.33	27.58
3	9.15	27.09
4	8.92	26.61
5	8.64	26.49
6	8.66	26.23
8	8.62	25.51
10	8.63	25.50
20	8.64	25.54
30	8.60	25.42
40	8.66	25.25
50	8.58	25.03
60	8.61	25.21
70	8.61	25.25
80	8.60	25.21
90	8.61	25.14
100	8.58	25.13
200	8.56	25.15
300	8.58	25.06
400	8.58	25.02
500	8.57	25
600	8.57	25



Support Vector Machine – Monthly

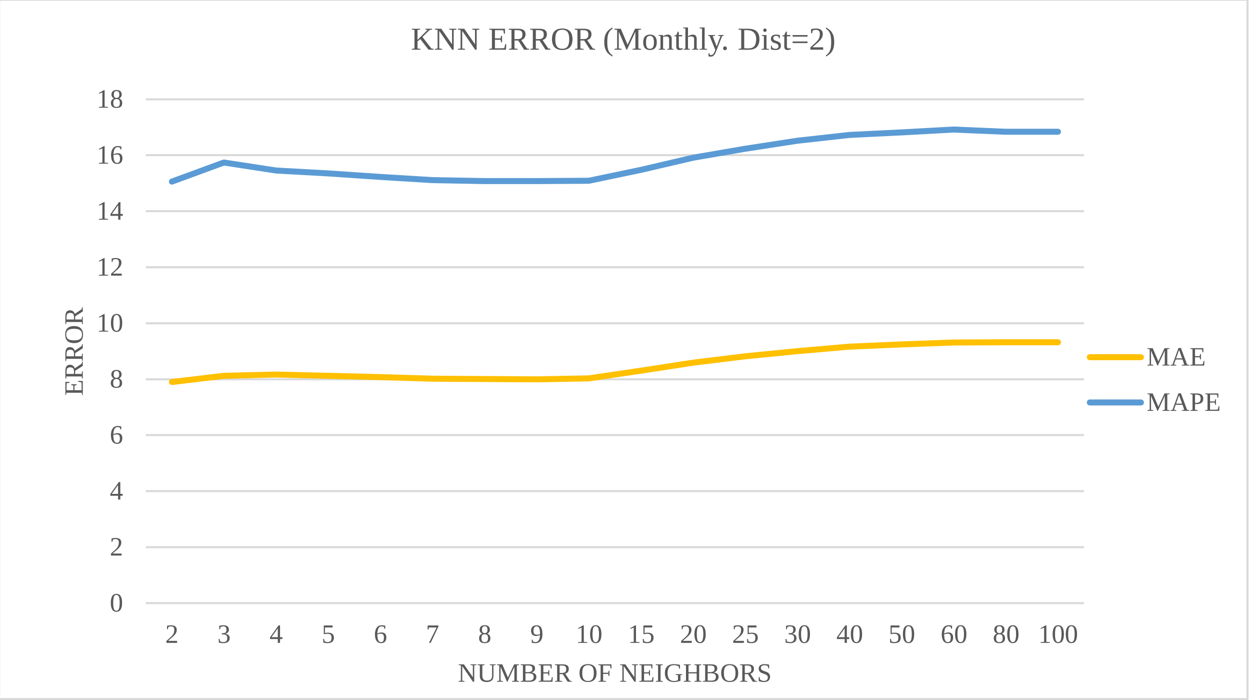
COST/ SIGMA	**0.001**	**0.01**	**0.1**	**1**	**10**	**100**
0.001	MAE: 8.48 MAPE: 15.23	MAE: 8.48 MAPE: 15.23	MAE: 8.48 MAPE: 15.23	MAE: 8.48 MAPE: 15.23	MAE: 8.48 MAPE: 15.23	MAE: 8.48 MAPE: 15.23
0.01	MAE: 8.48 MAPE: 15.24	MAE: 8.5 MAPE: 15.27	MAE: 8.51 MAPE: 15.28	MAE: 8.51 MAPE: 15.28	MAE: 8.51 MAPE: 15.28	MAE: 8.51 MAPE: 15.28
0.1	MAE: 8.51 MAPE: 15.33	MAE: 8.59 MAPE: 15.59	MAE: 8.66 MAPE: 15.51	MAE: 8.65 MAPE: 15.5	MAE: 8.65 MAPE: 15.5	MAE: 8.65 MAPE: 15.5
1	MAE: 8.48 MAPE: 15.77	MAE: 9.23 MAPE: 17.76	MAE: 8.69 MAPE: 15.55	MAE: 8.73 MAPE: 15.62	MAE: 8.73 MAPE: 15.62	MAE: 8.73 MAPE: 15.62
10	MAE: 11.31 MAPE: 24.05	MAE: 9.34 MAPE: 18.43	MAE: 8.62 MAPE: 15.45	MAE: 8.67 MAPE: 15.54	MAE: 8.67 MAPE: 15.54	MAE: 8.67 MAPE: 15.54
100	MAE: 20.3 MAPE: 42.29	MAE: 9.34 MAPE: 18.43	MAE: 8.62 MAPE: 15.45	MAE: 8.67 MAPE: 15.54	MAE: 8.67 MAPE: 15.54	MAE: 8.67 MAPE: 15.54

Support Vector Machine - Daily

COST/ SIGMA	**0.001**	**0.01**	**0.1**	**1**	**10**	**100**
0.001	MAE: 9.68 MAPE: 25.47	MAE: 9.63 MAPE: 25.42	MAE: 9.68 MAPE: 25.45	MAE: 9.68 MAPE: 25.46	MAE: 9.68 MAPE: 25.46	MAE: 9.68 MAPE: 25.46
0.01	MAE: 9.67 MAPE: 25.51	MAE: 9.57 MAPE: 25.45	MAE: 9.64 MAPE: 25.39	MAE: 9.68 MAPE: 25.47	MAE: 9.68 MAPE: 25.47	MAE: 9.68 MAPE: 25.47
0.1	MAE: 9.66 MAPE: 25.56	MAE: 9.3 MAPE: 25.49	MAE: 9.38 MAPE: 25.02	MAE: 9.72 MAPE: 25.5	MAE: 9.72 MAPE: 25.5	MAE: 9.72 MAPE: 25.5
1	MAE: 9.63 MAPE: 25.5	MAE: 8.61 MAPE: 27.26	MAE: 8.44 MAPE: 24.38	MAE: 9.82 MAPE: 25.59	MAE: 9.82 MAPE: 25.6	MAE: 9.82 MAPE: 25.6
10	MAE: 9.23 MAPE: 26.78	MAE: 14.63 MAPE: 44.04	MAE: 8.58 MAPE: 25.77	MAE: 9.91 MAPE: 25.68	MAE: 9.92 MAPE: 25.69	MAE: 9.92 MAPE: 25.69
100	MAE: 10.32 MAPE: 33.21	MAE: 23.99 MAPE: 68.89	MAE: 8.58 MAPE: 25.77	MAE: 9.91 MAPE: 25.68	MAE: 9.92 MAPE: 25.69	MAE: 9.92 MAPE: 25.69

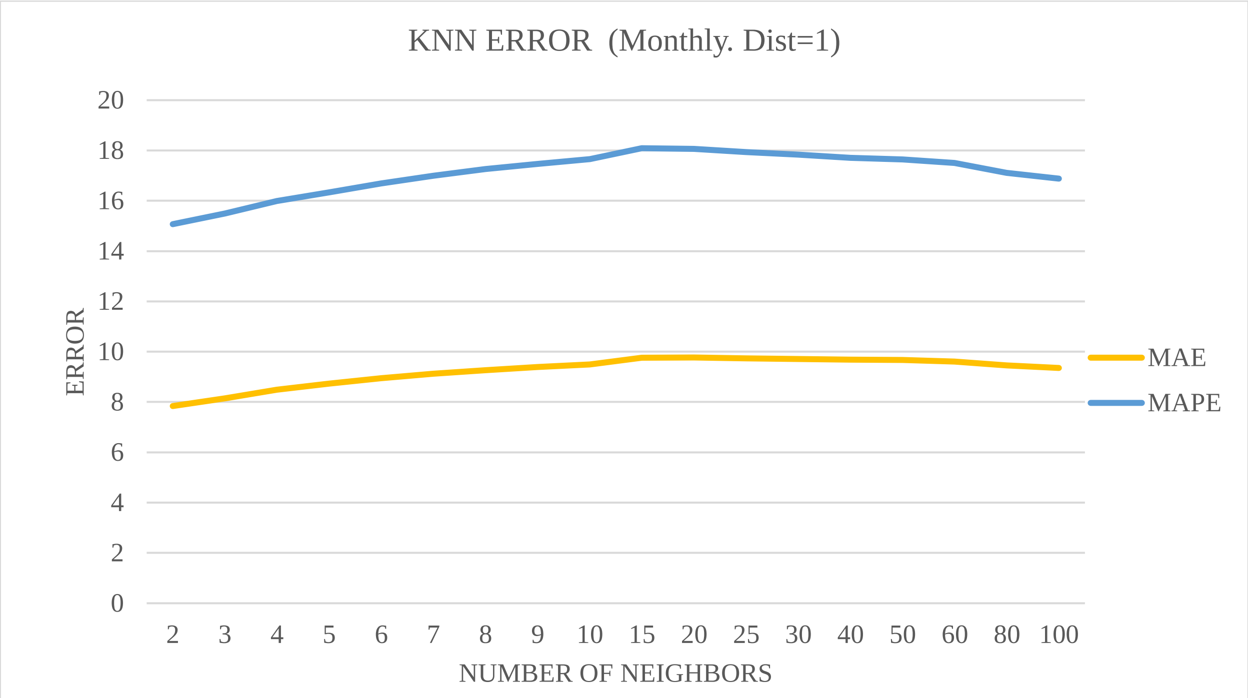
K-Nearest Neighbor – Monthly (Euclidean Distance: dist_power = 2)

DISTANCE	***#NEIGHBORS***	***MAE***	***MAPE***
2	2	7.90	15.06
2	3	8.12	15.74
2	4	8.17	15.46
2	5	8.12	15.35
2	6	8.07	15.23
2	7	8.02	15.11
2	8	8.01	15.08
2	9	8.00	15.08
2	10	8.03	15.09
2	15	8.30	15.48
2	20	8.59	15.91
2	25	8.82	16.24
2	30	9.00	16.52
2	40	9.16	16.73
2	50	9.24	16.82
2	60	9.31	16.92
2	80	9.32	16.84
2	100	9.32	16.84



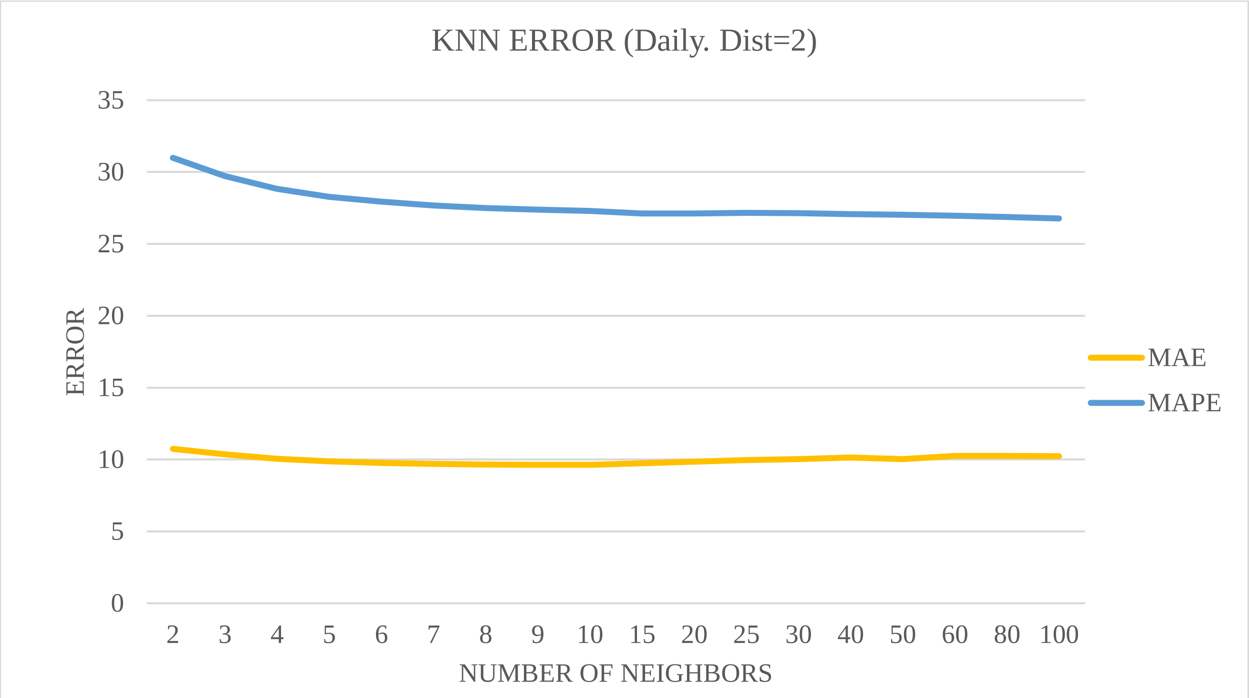
K-Nearest Neighbor – Monthly (ManhattanDistance: dist_power = 1)

DISTANCE	*#NEIGHBORS*	*MAE*	*MAPE*
1	2	7.84	15.07
1	3	8.15	15.49
1	4	8.49	15.99
1	5	8.73	16.34
1	6	8.95	16.69
1	7	9.12	17
1	8	9.27	17.26
1	9	9.39	17.47
1	10	9.50	17.66
1	15	9.76	18.09
1	20	9.77	18.07
1	25	9.74	17.94
1	30	9.71	17.83
1	40	9.68	17.71
1	50	9.67	17.65
1	60	9.61	17.51
1	80	9.45	17.11
1	100	9.35	16.88



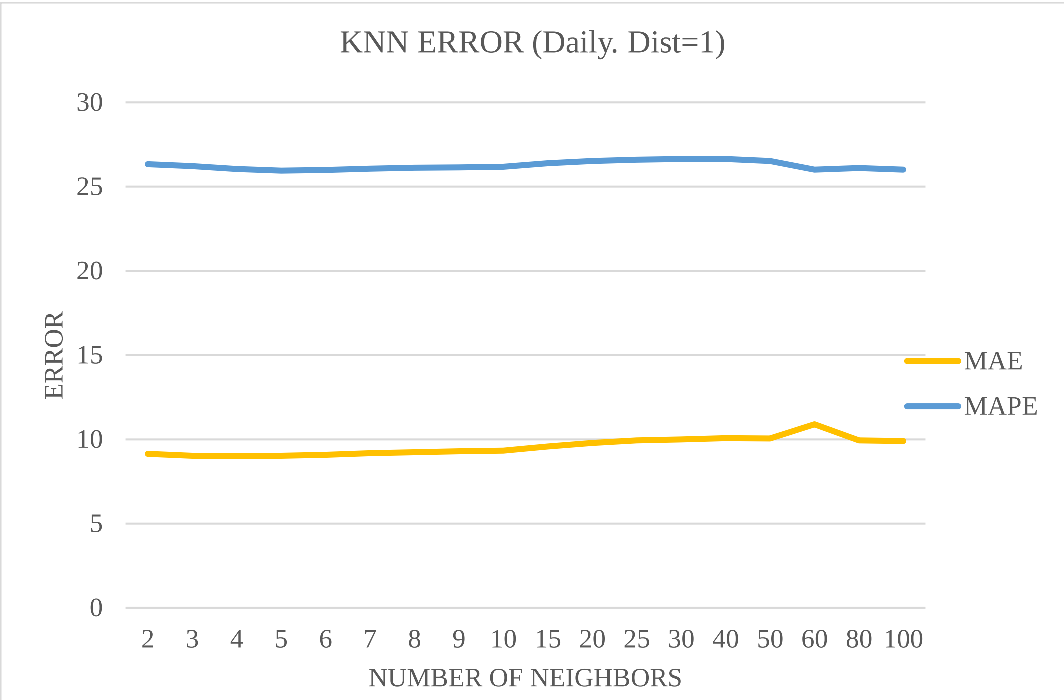
K-Nearest Neighbor – Daily (Euclidean Distance: dist_power = 2)

DISTANCE	***#NEIGHBORS***	***MAE***	***MAPE***
2	2	10.74	30.99
2	3	10.36	29.73
2	4	10.06	28.84
2	5	9.87	28.27
2	6	9.77	27.93
2	7	9.69	27.68
2	8	9.65	27.50
2	9	9.63	27.38
2	10	9.63	27.29
2	15	9.73	27.11
2	20	9.85	27.12
2	25	9.96	27.16
2	30	10.03	27.15
2	40	10.13	27.08
2	50	10.02	27.02
2	60	10.24	26.95
2	80	10.25	26.87
2	100	10.23	26.77



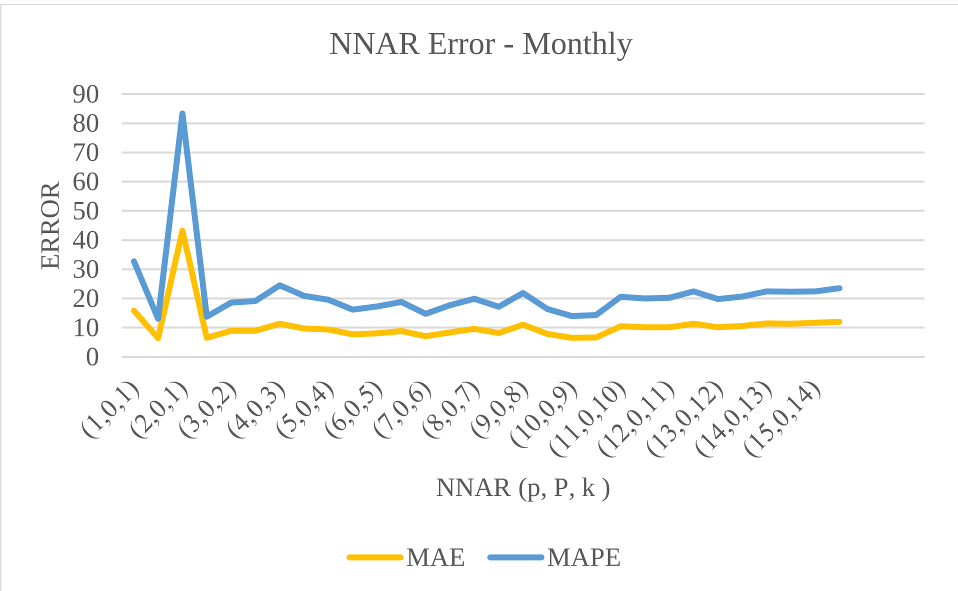
K-Nearest Neighbor – Daily (ManhattanDistance: dist_power = 1)

DISTANCE	***#NEIGHBORS***	***MAE***	***MAPE***
1	**2**	9.13	26.33
1	3	9.03	26.21
1	4	9.01	26.04
1	5	9.03	25.95
1	6	9.08	25.98
1	7	9.17	26.07
1	8	9.23	26.13
1	9	9.28	26.15
1	10	9.32	26.18
1	15	9.57	26.39
1	20	9.79	26.52
1	25	9.93	26.59
1	30	10.00	26.64
1	40	10.06	26.64
1	50	10.05	26.52
1	60	10.89	26.00
1	80	9.93	26.11
1	100	9.89	26.00



Neural Networks – Monthly Data

p	P	K	MAE	MAPE
1	0	1	15.81	32.73
1	0	2	6.42	13.06
2	0	1	43.24	83.35
2	0	2	6.54	13.74
3	0	2	8.95	18.55
3	0	3	8.95	19.19
4	0	3	11.39	24.46
4	0	4	9.67	20.89
5	0	4	9.34	19.59
5	0	5	7.79	16.22
6	0	5	8.04	17.25
6	0	6	8.84	18.8
7	0	6	7.07	14.77
7	0	7	8.4	17.76
8	0	7	9.6	19.88
8	0	8	8.13	17.2
9	0	8	11.02	21.89
9	0	9	7.85	16.35
10	0	9	6.56	13.94
10	0	10	6.69	14.29
11	0	10	10.49	20.58
11	0	11	10.15	19.98
12	0	11	10.18	20.2
12	0	12	11.35	22.46
13	0	12	10.11	19.84
13	0	13	10.6	20.71
14	0	13	11.45	22.41
14	0	14	11.39	22.33
15	0	14	11.7	22.4
15	0	15	12.02	23.51



Neural Networks – Daily Data

p	P	K	MAE	MAPE
1	1	11	13.13	32.32
1	2	11	14.25	33.02
2	1	11	14.27	32.91
2	2	11	13.51	32.91
3	2	11	12.11	30.75
3	3	11	8.79	24.99
4	3	11	12.9	31.11
4	4	11	14.27	34.04
5	4	11	11.05	29.05
5	5	11	11.91	30.23
6	5	11	10.14	27.4
6	6	11	11.72	30.42
7	6	11	11.72	30.42
7	7	11	8.39	25.24
8	7	11	9.84	26.35
8	8	11	8.71	24.72
9	8	11	11.83	30.71
9	9	11	10.75	28.83
10	9	11	9.79	26.27
10	10	11	8.43	25.57
11	10	11	8.01	24.58
11	11	11	16.58	37.72
12	11	11	8.63	25
12	12	11	9.91	27.15